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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

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VOLUME LII — SEPTEMBER 1920 — NUMBER 2

RADIATION PRESSURE ON ELECTRONS AND ATOMS

By LEIGH PAGE

ABSTRACT

Radiation pressure on electrons and atoms.—Assuming the classical electrodynamic equations, the author shows (1) that the radiation pressure on a Lorentz electron is three times the pressure on a perfectly reflecting plane normal to the radiation, and (2) that the mean pressure on a large number of linear simple harmonic atomic oscillators oriented at random is chiefly a resonance pressure. It is a function of the ratio of wave-length to atomic diameter and of the natural frequency of the oscillators, such that the pressure of solar radiation at the surface of the sun is greater than the gravitational attraction for the lighter atoms if they have one or more resonant frequencies between wave-lengths $0.3\ \mu$ and $10\ \mu$. These calculations show that classical electrodynamics is competent to *explain observed phenomena of radiation pressure on attenuated gases*. Contrary conclusions were reached by previous authors mainly as a result of neglecting resonance pressure.

In a recent paper[1] Megh Nad Saha, in speaking of the classical theory of radiation pressure, makes the statement: "If the particle be too small, it is no longer capable of acting as a barrier to the advancing light-waves, and consequently experiences no radiation pressure." As the present author believes that this statement involves a misconception of one of the important consequences of classical electrodynamics, it seems worth while to present a detailed calculation of the radiation pressure on electrons and atoms.

[1] *Astrophysical Journal*, **50**, 220, 1919.

The radiation pressure on small spheres of gross matter having radii comparable with the wave-length of the incident radiation has been computed by Schwarzschild,[1] Nicholson,[2] Proudman,[3] and Debye,[4] and these authors find that the pressure rises to a maximum, and then falls off to zero as the radius of the obstructing particle diminishes. With the exception of the last named, however, they do not seem to have considered resonant pressure, which must have the predominant effect in the case of single atoms or molecules, and which is referred to in R. W. Wood's *Optics*.[5]

RADIATION PRESSURE ON AN ISOLATED ELECTRON

Consider first the case of a single electron under the influence of a train of plane polarized waves traveling in the X direction. If the electric vector is parallel to the Y axis,

$$E_y = A \cos(\sigma x - \omega t),$$
$$H_z = A \cos(\sigma x - \omega t),$$

the other components of E and H vanishing. In order to prevent motion in the direction of propagation of the wave, apply an electrostatic field E_x parallel to the X axis of sufficient intensity to balance the average radiation pressure. If the displacements of the electron parallel to the co-ordinate axes are denoted by ξ, η, ζ, its equations of motion in Heaviside-Lorentz rational units are

$$E_x + \overline{\frac{1}{c}\frac{d\eta}{dt} H_z} = O,$$

$$E_y = M\frac{d^2\eta}{dt^2} - N\frac{d^3\eta}{dt^3},$$

where

$$M \equiv \frac{m}{e}, \qquad N \equiv \frac{e}{6\pi c^3}.$$

[1] *Sitzungsberichte der Math. Phys. Classe zu München*, 293, 1901–1902.

[2] *Monthly Notices, R.A.S.*, **70**, 544, 1910.

[3] *Ibid.*, **73**, 535, 1913.

[4] *Annalen der Physik*, **30**, 57, 1909.

[5] *Physical Optics*, 2d ed., p. 613.

Solving, and taking into account the fact that $N\omega$ is very small compared to M, the average force due to radiation pressure after a steady state has been reached is found to be

$$F=\overline{\frac{e}{c}\frac{d\eta}{dt}H_z}$$

$$=\tfrac{1}{2}A^2\frac{e}{c}\frac{N}{M^2}.$$

As the average intensity I of the radiation incident on the electron is

$$I=\tfrac{1}{2}A^2c,$$

and, in the case of the Lorentz electron of radius a,

$$M=\frac{e}{6\pi ac^2},$$

it follows that

$$F=6\pi a^2\frac{I}{c},$$

or the force per unit area of the electron's cross-section is

$$p=6\frac{I}{c}. \tag{1}$$

When a train of waves is reflected normally from an infinite perfectly conducting surface, the radiation pressure is

$$p'=2\frac{I}{c}, \tag{2}$$

which is only one-third of that on the isolated electron considered above.

The rate of absorption of energy by the single electron under consideration is found by the usual methods to be

$$U=6\pi a^2 I,$$

or the energy absorbed per unit area of the electron's cross-section per unit time is

$$u = 6I. \tag{3}$$

This energy is re-radiated so that as much leaves the electron in the positive as in the negative X direction. Therefore the emitted radiation exerts no reaction on the electron, and the force on the latter, according to the principle of conservation of momentum, should be equal to the rate of loss of momentum by the incident beam. This gives the expression (1) for the radiation pressure already obtained by a more direct method.

RADIATION PRESSURE ON AN ATOMIC OSCILLATOR

Consider next a linear simple harmonic oscillator, of which the vibrating part is an electron constrained to move in the Y direction only. The equation of motion is

$$E_y = K\eta + M\frac{d^2\eta}{dt^2} - N\frac{d^3\eta}{dt^3},$$

and the solution for the steady state, which was used by Planck many years ago in connection with the derivation of the law of black radiation, is

$$\eta = \frac{A}{N\omega^3}\sin\delta\cos(\sigma x - \omega t + \delta),$$

where

$$\delta \equiv \tan^{-1}\frac{N\omega^3}{M(\omega_o^2 - \omega^2)}, \qquad \omega_o \equiv \sqrt{\frac{K}{M}}.$$

The force due to radiation pressure is easily found to be

$$F = \overline{\frac{e}{c}\frac{d\eta}{dt}H_z}$$

$$= \frac{3}{2\pi}\lambda^2\frac{I}{c}\sin^2\delta. \tag{4}$$

This force[1] is greatest when sin δ equals unity, which occurs when the natural period of the oscillator is the same as that of the

[1] Cf. Debye, *Annalen der Physik*, **30**, 101, 1909.

incident radiation. Since the wave-length λ is in general much greater than the linear dimensions of an atomic oscillator, the resonant radiation pressure is considerably greater than that on a gross perfectly reflecting surface. The reader may easily show that the same relation exists between the rate of absorption of energy and the radiation pressure as in the case of an isolated electron. In fact, this relation may be proved in a quite general manner.

If the axis of the linear oscillator makes an angle θ with the X axis, and if the plane determined by these two lines makes an angle ϕ with the $X\ Y$ co-ordinate plane, the displacements of the oscillator are

$$\xi=\frac{A\sin\theta\cos\theta\cos\phi}{N\omega^3}\sin\delta\cos(\sigma x-\omega t+\delta),$$

$$\eta=\frac{A\sin^2\theta\cos^2\phi}{N\omega^3}\sin\delta\cos(\sigma x-\omega t+\delta),$$

$$\zeta=\frac{A\sin^2\theta\sin\phi\cos\phi}{N\omega^3}\sin\delta\cos(\sigma x-\omega t+\delta),$$

and the components of the force due to radiation pressure are

$$F_x=\frac{3}{2\pi}\lambda^2\frac{I}{c}\sin^2\theta\cos^2\phi\sin^2\delta,$$

$$F_y=-\frac{3}{2\pi}\lambda^2\frac{I}{c}\sin\theta\cos\theta\cos\phi\sin^2\delta,$$

$$F_z=0.$$

If the axes of a number of oscillators point indiscriminately in all directions, the average force is seen to be entirely in the direction of propagation of the radiation, and equal in amount to

$$F=\frac{\lambda^2}{2\pi}\frac{I}{c}\sin^2\delta, \tag{5}$$

which is one-third the value of the force on an oscillator whose axis is parallel to the direction of the electric vector, as might have been expected.

Next consider the case of an oscillator of natural frequency ν_0 exposed to general radiation propagated along the X axis. If

$$I_\nu d\nu$$

is the intensity of radiation of frequencies between ν and $\nu+d\nu$, equation (5) must be replaced by

$$\begin{aligned} F &= \frac{c}{2\pi}\int_0^\infty I_\nu \sin^2\delta \frac{d\nu}{\nu^2} \\ &\doteq \frac{cI_\nu}{2\pi\nu_0^2}\int_0^\infty \sin^2\delta\, d\nu \\ &\doteq \tfrac{1}{2}\pi a I_{\nu 0}, \end{aligned} \tag{6}$$

where a is the radius of the vibrating electron. The radiation pressure, as was to be expected, depends on the intensity of that portion of the incident radiation which has a frequency equal to the natural frequency of the oscillator.

If the distribution of energy in the incident radiation is that of the normal radiation spectrum,

$$I_\nu = \frac{15}{\pi^4}\frac{h}{kT}\frac{x^3}{e^x-1} I,$$

where

$$x \equiv \frac{h\nu}{kT}.$$

Therefore

$$F = \frac{15a}{2\pi^3}\frac{hc}{kT}\frac{x^3}{e^x-1}\frac{I}{c}. \tag{7}$$

If the radiation from the sun has a temperature of about 6000°,

$$\frac{hc}{kT} = 2.5(10)^{-4} \text{ cm},$$

and, as the radius a of the electron is about $2(10)^{-13}$ cm,

$$F \doteq 4\pi(10)^{-18} \text{ cm}^2 \frac{x^3}{e^x-1}\frac{I}{c}. \tag{8}$$

Therefore, if the radius of the atomic vibrator is taken as $(10)^{-8}$ cm, the radiation pressure per unit area of the atom's cross-section is

$$p = 0.04 \frac{I}{c}\left[\frac{x^3}{e^x - 1}\right]. \tag{9}$$

If the function

$$f(x) \equiv \frac{x^3}{e^x - 1}$$

is evaluated for different values of the wave-length λ, the following results are obtained:

λ	$f(x)$
$2.8\ (10)^{-4}$ cm	0.5
1.6	1.0
0.9	1.4 (max)
0.6	1.1
0.4	0.5

Therefore the pressure of solar radiation is greatest on an atom which has a resonant frequency in the infra-red near to wave-length 9000 A, and the radiation pressure has a magnitude greater than one-third of this maximum value if the atom has a resonant frequency anywhere between 4000 A and 28,000 A, a range of nearly three octaves. If an atom has more than one resonant frequency in this interval, the pressure is correspondingly greater.

That the value of the pressure, as given by (9), is small compared to that on gross matter is due to the somewhat arbitrary manner in which it has been defined. More significant results are obtained by comparing the force (8) due to radiation pressure with gravitational attraction.

If the sun radiates as an ideal black body at temperature 6000°, the effective intensity of radiation[1] at its surface in the direction away from its center is given by

$$\frac{I}{c} = 1.5 \frac{\text{gm}}{\text{cm sec}^2},$$

[1] This value is somewhat smaller than the $2.7 \frac{\text{gm}}{\text{cm sec}^2}$ computed on other assumptions by Arrhenius and mentioned by Nicholson.

and hence the force on an atom due to radiation pressure is

$$F = 1.9\,(10)^{-17}\left[\frac{x^3}{e^x - 1}\right] \text{ dynes.}$$

The gravitational attraction, however, on an atom of atomic mass A is

$$G = 4.4(10)^{-20}\,A \text{ dynes.}$$

Hence

$$\frac{F}{G} = \frac{430}{A}\left[\frac{x^3}{e^x - 1}\right].$$

So, for atomic masses of the order of 20 or less, the repulsion due to radiation pressure will overbalance gravitational attraction, provided

$$\frac{x^3}{e^x - 1} > 0.05,$$

that is, provided a single resonant frequency lies between wavelengths 2500 A and 100,000 A. The radiation pressure is greatest for a resonant frequency at a wave-length of 9000 A, in which case the repulsive force due to radiation may be thirty or more times as great as the attraction due to gravitation.

The preceding calculations seem to show that the classical electrodynamics is quite competent to explain the observed phenomena of radiation pressure on attenuated gases (such as may constitute comets' tails), and that it is not necessary to resort to assumptions involving absorption of "light-quanta."

Mr. N. I. Adams has kindly verified the analysis appearing in this paper.

NOTE.—Since writing this paper, the author's attention has been called to a paper by Gouy (*Comptes Rendus*, **157**, 186, 1913) in which similar conclusions have been reached regarding resonance pressure on atomic vibrators.

SLOANE PHYSICS LABORATORY
YALE UNIVERSITY
June 26, 1920

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

SEVENTEENTH PAPER: MISCELLANEOUS RESULTS

By HARLOW SHAPLEY

ABSTRACT

Discovery and position co-ordinates of 88 variable stars in globular clusters N.G.C. 5024, 6293, 6553, 6779, 6864, 6981, 7006, and 7492 are collected in Table II.

Absolute visual magnitudes of forty globular clusters are given in Table III. More than half lie within the interval -8.3 to -9.3, the mean being -8.8 ± 0.5. This remarkable agreement in absolute magnitude enables parallaxes to be estimated from the apparent brightness with a probable error of less than 25 per cent. The *total light-emission of an average cluster* is about 275,000 times that of the sun.

Galactic cluster N.G.C. 7789 is found from the count of 13,500 stellar images to contain about 1100 stars with magnitudes ranging from 13 to 20. The probable linear diameter is 20 parsecs.

Theoretical conclusions regarding Cepheid variables.—Assuming the average heat content per unit mass and therefore the ratio of mass to radius to be the same for all these variables, the author derives a theoretical period-luminosity curve which fits the observations well if the mean atomic weight has the value indicated by other considerations, viz., between 2.5 and 3.0. This agreement lends support to the theory and to the conclusion that only giant stars with a particular ratio of mass to radius become Cepheid variables.

Globular cluster N.G.C. 7006 is so distant that the faintest stars recorded on the plate, magnitude 20, are probably a hundred times as bright as our sun. The star count indicates that the cluster is similar in constitution to nearer ones.

Density of stars in galactic longitude $+32°$ and latitude $-20°$, to photographic magnitude 20, is about 23,000 per square degree.

Following the plan of the fourteenth paper, the present contribution includes five notes on various aspects of the investigation of clusters. Three of these contain new observational results; the other two deal with material already published. The subjects are:

I. The discovery of 88 variable stars in eight globular clusters.

II. The total light-emission of an average globular cluster and the absolute visual magnitudes of forty systems.

III. The number of stars in N.G.C. 7789, an open cluster of the galactic system.

[1] *Contributions from the Mount Wilson Observatory*, No. 190.

IV. Evidence that a giant star must have a particular ratio of mass to radius before it can become a Cepheid variable (a theoretical period-luminosity law).

V. A further investigation of the most distant globular cluster now known, N.G.C. 7006.

I. POSITION CO-ORDINATES OF NEW VARIABLE STARS

Upon the request of the late Professor Pickering we have undertaken at Mount Wilson to supplement the systematic discovery and study of variable stars in globular clusters (carried on at Harvard under Professor Bailey's supervision) by examining those faint and highly condensed systems which cannot be satisfactorily studied with the Harvard telescopes. Also, for the brighter clusters, special series of plates have been made and sent to Professor Bailey to supplement the Harvard material in the study of the light-curves of cluster variables; and some progress has been made in examining the fainter stars in bright clusters for evidence of variability.

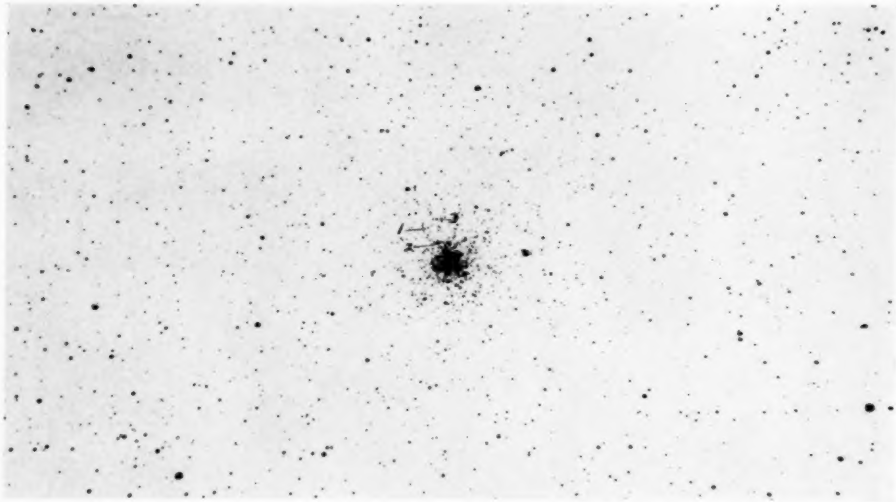
The typical variable in a globular cluster is the short-period Cepheid. Through the remarkable law that the length of the period is uniquely related to the absolute brightness, these variables are of paramount importance in determining the distances and space distribution of clusters. Particularly for the most remote clusters it is important to check by means of Cepheid variables the distances already derived through measures of angular diameter and the magnitudes of the brightest stars.

Positions of 23 new variables near the center of Messier 3 are given in *Mount Wilson Contr.*, No. 91, 1914. The variable stars found by Miss Davis on my photographs of clusters were reported in *Publications of the Astronomical Society of the Pacific*, **29**, 210, 260, 1917; and the positions and magnitudes of the 28 variables found in Messier 68 by Miss Ritchie are included in the general discussion of that cluster, *Mount Wilson Contr.*, No. 175, 1919.[1] The clusters in which other variables have been found by Miss

[1] Positions of 19 of these variables are also published in *Publications of the Astronomical Society of the Pacific*, **31**, 226, 1919. A faint variable in Messier 9 has also been found on Mount Wilson plates, *ibid.*, **28**, 282, 1916.

PLATE IV

a



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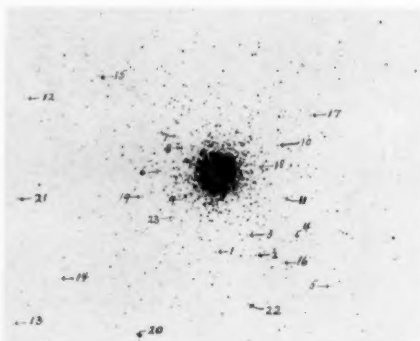

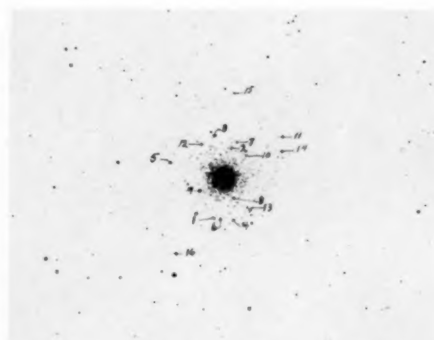

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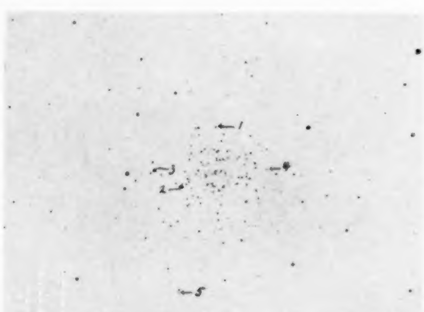


e

VARIABLE STARS IN CLUSTERS

a. N.G.C. 6779 = Messier 56
b. N.G.C. 5024 = Messier 53
c. N.G.C. 6864 = Messier 75
d. N.G.C. 7492
e. N.G.C. 6981 = Messier 72

Scale: 1 mm = 16″

Ritchie are listed in Table I, taking the co-ordinates and adopted values of the parallax from *Mount Wilson Contr.*, Nos. 152 and 161.

The equatorial co-ordinates of the new variables, in seconds of arc from an adopted center, are given in Table II; the orientation, however, is not exact. An asterisk after the number in the first column indicates that the star is suspected of variation, but the plates now available are insufficient for a definite proof.

On the accompanying reproduction of photographs of five clusters (Plate IV) the variable stars are indicated by arrows.

TABLE I

N.G.C.	Messier	Right Ascension 1900	Declination 1900	Parallax	Number of Variables	Number Suspected of Variation
5024	53	13^h 8^m0	$+18°42'$	$0''000053$	23	....
6293	....	17 4.0	−26 26	0.000040	3	....
6553	....	18 3.2	−25 56	0.000031	....	2
6779	56	19 12.7	+30 0	0.000040	1	2
6864	75	20 0.2	−22 12	0.000022	11	5
6981	72	20 48.0	−12 55	0.000034	31	3
7006	....	20 56.8	+15 48	0.000015	2	....
7492	....	23 3.1	−16 10	0.000035	1	4

Nearly all of the variables in these clusters appear to have periods of less than a day. This is particularly true for the 23 in Messier 53 and the 31 in Messier 72; for both clusters the median magnitudes of the variables are closely grouped around a mean value, in agreement with the earlier results for Messier 3, 5, 15, 22, 68, and ω Centauri.

A study of the variable stars in Messier 75, at least the determination of their median magnitudes, will be of special interest because the cluster appears to be one of the most distant objects on record, $R=45{,}000$ parsecs. A photographic study of the variables in the faint cluster Messier 72 will be published as the next contribution of this series.

II. THE ABSOLUTE MAGNITUDES OF FORTY GLOBULAR CLUSTERS

The integrated visual magnitudes of forty globular clusters have been measured at Vienna by Holetschek.[1] Parallaxes have been published for all of them in earlier contributions of this series.

[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, **20**, 114, 1907.

TABLE II

Variable	x	y
N.G.C. 5024		
1	+0′ 9″.6	−2′51″.0
2	−1 18.0	−3 3.6
3	−1 0.6	−2 18.0
4	−2 49.5	−2 36.6
5	−3 57.0	−4 18.0
6	+2 3.6	+0 13.5
7	+1 19.5	+1 23.5
8	+1 12.0	+1 0
9	+1 7.5	−0 40.5
10	−2 18.6	+0 54.0
11	−2 23.4	−0 58.5
12	+6 49.5	+3 7.5
13	+7 42.0	−4 59.7
14	+5 54.6	−3 27.0
15	+4 8.4	+3 48.0
16	−2 16.5	−3 22.5
17	−3 34.5	+1 54.0
18	−1 36.0	+0 12.6
19	+2 45.6	−0 42.0
20	+3 8.4	−5 51.6
21	+7 17.4	−0 27.0
22	−0 53.4	−4 48.0
23	+1 36.0	−1 29.7
N.G.C. 6293		
1	+1 21.0	+0 49.5
2	−2 15.6	+1 4.5
3	+0 48.6	+0 18.6
N.G.C. 6553		
1*	+0 4.5	+0 0.3
2*	+0 50.4	−0 1.2
N.G.C. 6779		
1	+0 51.0	+1 15.6
2*	+0 21.0	+0 54.4
3*†	+0 33.0	+2 4.5
N.G.C. 6864		
1	+0 15.6	−1 23.4
2	−0 9.0	+0 54.0
3	+0 18.0	+1 25.5
4	−0 18.0	−1 24.6
N.G.C. 6864—*Continued*		
5	+1′48″.0	−0′36″.0
6	+0 8.4	−1 21.0
7	−0 24.6	+1 18.0
8	−0 13.5	−0 41.4
9	+0 45.6	−0 24.0
10*	−0 43.5	+0 50.4
11	+2 1.2	+1 24.0
12	+0 39.6	+1 15.0
13*	−0 53.4	−1 16.5
14*	−2 4.5	+0 51.0
15*	−0 19.5	+3 3.0
16*	+1 50.4	−2 43.5
N.G.C. 6981		
1	+0 43.5	−0 54.0
2	+1 39.0	+3 14.4
3	−0 52.5	−0 58.5
4	−1 46.5	+0 37.5
5	−0 38.4	−0 21.6
6*	+1 18.0	+1 18.6
7	−0 3.6	+0 55.5
8	−0 6.6	+1 29.4
9	+0 11.4	+0 50.4
10	−0 48.6	−1 13.5
11	+0 57.0	−0 36.6
12	+0 9.0	−0 21.6
13	+0 13.5	+0 17.4
14	−0 13.5	+0 36.0
15	−1 4.5	−0 21.0
16	−0 4.5	−0 19.5
17	+0 3.6	−0 43.5
18	−0 26.4	−0 37.5
19*	+0 3.0	+1 52.5
20	−0 54.6	+0 15.0
21	−1 22.5	+0 12.6
22	−1 53.4	+0 1.5
23	−1 39.0	+1 56.4
24	−0 15.6	−0 24.0
25	−2 13.5	+1 7.5
26	−1 31.5	−0 45.0
27	+3 29.4	−3 54.0
28	+1 5.4	+1 21.0
29	−0 36.0	−0 52.5
30	+1 11.4	−1 37.5
31	+0 5.4	+0 36.6
32	−2 18.0	−0 42.0
33*	−0 2.4	−1 0.6
34	+0 6.0	+0 7.5

* Suspected.

† Found independently by Miss Davis, *Publications of the Astronomical Society of the Pacific*, **29**, 210, 1917, where the position is referred to a different origin.

TABLE II—*Continued*

Variable	x	y	Variable	x	y
N.G.C. 7006			N.G.C. 7492		
1.	−0′22″.5	+0′36″.6	1.	+0′ 1″.2	+1′36″.6
2.	−0 37.5	−0 39.6	2*.	+1 12.0	−0 27.0
			3*.	+2 22.5	−0 7.5
			4*.	+1 51.0	+0 4.5
			5*.	+1 21.0	−4 17.4

These values of distance and apparent brightness permit the computation of the integrated absolute magnitudes presented in Table III.

All but two clusters fall within the magnitude interval from −6.8 to −9.7, and more than half within the interval −8.3 to −9.3. This remarkable similarity in total light-emission again

TABLE III

ABSOLUTE MAGNITUDE FOR GLOBULAR CLUSTERS

N.G.C.	Abs. Mag.	N.G.C.	Abs. Mag.	N.G.C.	Abs. Mag.	N.G.C.	Abs. Mag.
1904. . . .	−9.0	6121. . . .	−8.5	6287. . . .	−9.0	6712. . . .	− 8.6
4147. . . .	−9.2	6171. . . .	−7.0	6293. . . .	−8.6	6760. . . .	− 8.1
4590. . . .	−7.8	6205. . . .	−9.4	6333. . . .	−9.7	6779. . . .	− 8.7
5024. . . .	−8.6	6218. . . .	−8.7	6341. . . .	−9.2	6864. . . .	−10.3
5272. . . .	−9.1	6229. . . .	−9.6	6356. . . .	−9.4	6934. . . .	− 8.6
5466. . . .	−8.0	6235. . . .	−8.8	6402. . . .	−9.0	6981. . . .	− 7.8
5634. . . .	−7.7	6254. . . .	−8.5	6626. . . .	−8.4	7006. . . .	− 9.4
5897. . . .	−5.7	6266. . . .	−8.9	6637. . . .	−7.6	7078. . . .	− 9.6
5904. . . .	−8.8	6273. . . .	−9.2	6656. . . .	−8.4	7089. . . .	− 9.2
6093. . . .	−8.7	6284. . . .	−8.3	6681. . . .	−6.8	7099. . . .	− 7.7

calls attention to the possibility of using apparent brightness as a measure of distance.[1] We have here, in fact, an independent method of determining relative parallaxes of clusters, for although the apparent magnitudes of Cepheids or of bright stars are indirectly involved in computing the absolute magnitudes of Table III, it should be specially noted that the adopted value of the distance of a cluster is not dependent on the apparent brightness of its

[1] The possibility was first noted in *Mt. Wilson Contr.*, No. 115, and discussed further in *Mt. Wilson Contr.*, No. 161, section vi; *Astrophysical Journal*, **50**, 107, 1919.

stars as a whole. The 25 brightest stars, for instance, contribute scarcely 2 per cent to the total light of a cluster.

From Table III we may derive the following values of the mean absolute magnitudes and of the average deviation from the mean:

	Number	Absolute Magnitude
All clusters	40	-8.59 ± 0.63
$\pi < 0\overset{''}{.}00005$	19	-8.81 ± 0.56
$\pi \geq 0.00005$	21	-8.39 ± 0.70
North of declination $-20°$	23	-8.78 ± 0.54
$\pi < 0\overset{''}{.}00005$	12	-8.82 ± 0.57
$\pi \geq 0.00005$	11	-8.74 ± 0.51

There is no indication that distant and nearby clusters differ in total luminosity.

Since the magnitude estimates for clusters south of declination $-20°$ are probably of lower weight than for clusters observable at small distance from the zenith, the average visual absolute magnitude of a globular cluster may be accepted as -8.8 ± 0.5 until further observation justifies revision of the data for distance or apparent magnitude.

By taking -8.8 as the mean value of the absolute visual magnitude, the distance of any globular cluster can be computed, from the apparent magnitude of the system as a whole, with an average probable error of less than 25 per cent. The relation connecting distance in parsecs, r, with apparent visual magnitude, m, is

$$\log r = 0.2\ (m + 13.8).$$

The total light-emission of an average globular cluster is, according to the foregoing result, 275,000 times that of the sun, and its energy of radiation is 10^{39} ergs per second.

III. ON THE DIMENSIONS AND STELLAR CONTENT OF N.G.C. 7789

Although the northern cluster N.G.C. 7789, in galactic latitude $-6°$, is well outside the region of the sky where globular clusters are found, its richness in faint stars suggests that it may be a type intermediate between the loosest of globular clusters and the most

populous of bright open systems. The hypothesis that these two main types represent different stages in the evolution of a stellar group makes the richest open systems of particular interest.

An hour's exposure on a fast plate with the 60-inch reflector demonstrates that N.G.C. 7789 is not globular, as it is not closely condensed toward the center, and the number of stars does not increase with decreasing brightness. The angular diameter is approximately 20′. Since the distance of the group has been estimated provisionally as 3300 parsecs,[1] the linear diameter appears to be only 20 parsecs, possibly one-sixth the average value for globular clusters.

On a series of five good photographs, all centered on the cluster, Miss Mayberry has counted the number of stars in a region 22′ by 28′. The exposure times range from one minute to one hour, and the total number of images counted is 13,500. An analysis of the star counts, which were recorded for every fourth of a square minute of arc on all plates, yields the following results for the whole region:

Photographic Magnitude Interval	<16	16–17	17–18	18–19	19–20	Total
Number of cluster stars	347	210	155	266	126	1104
Number of field stars	592	256	666	1178	1475	4167
Ratio	0.59	0.82	0.23	0.23	0.09	....

While qualitatively dependable, high weight should not be given to the numerical values in this tabulation, because of difficulty in allowing for the photometric effect of distance from the center in the case of clusters of large angular diameter, and because of uncertainty in the magnitude limits. Apparently the cluster stars lie mainly between magnitudes 15 and 19. The brightest are of photographic magnitude 13, corresponding at the distance given above to the absolute magnitude +0.5.

IV. NOTE ON THE PERIOD-LUMINOSITY CURVE OF CEPHEID VARIATION

The total content of heat in a gaseous mass may be expressed as the sum of the kinetic energy of translation of the molecules (or

[1] *Mt. Wilson Communication*, No. 62; *Proceedings of the National Academy of Sciences*, **5**, 344, 1919.

particles of the gas), and the energy within the molecules. If the mass is a typical giant star, therefore, in a steady state, the total heat, H, is necessarily proportional to the gravitational potential; that is,

$$H \propto \frac{\mu^2}{R} ,$$

where μ is the mass and R the radius. The factor of proportionality involves the ratio of the specific heats which conceivably may vary with mass or linear dimensions, but as all the possible values for a gaseous star lie between $4/3$ and $5/3$, this ratio will be treated below as a constant. The *average* heat-content per unit of mass is

$$C = k_o \frac{\mu}{R} . \tag{1}$$

Let us suppose that C has the same constant value for all Cepheid variables and check this supposition against observation. Then since $\mu = 4\pi R^3 \bar{\rho}/3$, where $\bar{\rho}$ is the mean density, we have, upon substitution for R in (1),

$$\bar{\rho}\mu^2 = \text{a constant}; \tag{1'}$$

and since in a gaseous star the period, P, of a gravitational pulsation is inversely proportional to the square root of the mean density,[1]

$$\frac{\mu}{P} = \text{a constant}. \tag{2}$$

The relation of the masses of two gaseous giant stars to the difference in their absolute (bolometric) magnitude, M, may be written (from Eddington's theory)[2]

$$M_n - M_1 = 2.5 \log_{10} \frac{\mu_1(1-\beta_1)}{\mu_n(1-\beta_n)} , \tag{3}$$

[1] In his mathematical discussion of the Cepheid problem, Eddington deduced $P^2\rho f(\gamma) = a$ *constant*, where the function of the ratio of specific heats, γ, changes slightly with the mass because of changes in the elastic constants.

[2] *Astrophysical Journal*, **48**, 205, 1918; cf. also Jeans, *Problems of Cosmogony and Stellar Dynamics*, Cambridge, England (1919), chap. viii.

where β, the ratio of the force of the ordinary gas pressure to gravitation, is given by

$$\frac{1-\beta}{\beta^4}=0.0026\ m^4\mu^2. \tag{4}$$

The unit of μ is the solar mass; and m, the average "molecular" weight, is expressed in terms of the hydrogen atom.

By adopting from data of observation the following values as defining a typical Cepheid variable

$$\left.\begin{array}{l} M_1=-2.2 \\ \mu_1=4.0 \\ P_1=5.4 \text{ days} \end{array}\right\} \tag{5}$$

we have

$$\log_{10} P=\log_{10}\mu+0.13. \tag{2'}$$

Taking $m=2$, 2.5, 3, and 4, I have used the preceding equations to compute μ, M, and the logarithm of P for selected intervals of β. The results are given in Table IV for $m=2.5$, in which case

$$M=-1.90-2.5\log_{10}\mu(1-\beta). \tag{3'}$$

Figure 1 compares the theoretical period-luminosity curve, computed on this assumption of an average heat-content, with all the available observations. Each plotted point represents the observed period and visual brightness of a Cepheid variable, the data being taken without change from my derivation of the period-luminosity curve in *Mount Wilson Contr.*, No. 151. The visual magnitude of the two brightest stars plotted should be decreased by 0.4 mag., according to a more recent examination of the color-period relation for Cepheids, thus bringing those two points in close agreement with the curve. Open circles indicate determinations of low weight.

For Cepheids with periods of less than three days the theoretical curve no longer fits the observations. This is to be expected, for the central density in such stars is commonly believed to be too great for further comparison with a gas. It has already been found that the cluster-type variables have anomalous physical

properties, in that stars of the same absolute magnitude and color differ in length of period by three to one.[1]

The observations are not so well represented by the theoretical period-luminosity curve when the average molecular weight is taken as 2 or 4; but with $m=3$ it is possible to represent the observed values nearly as well as with $m=2.5$, provided small

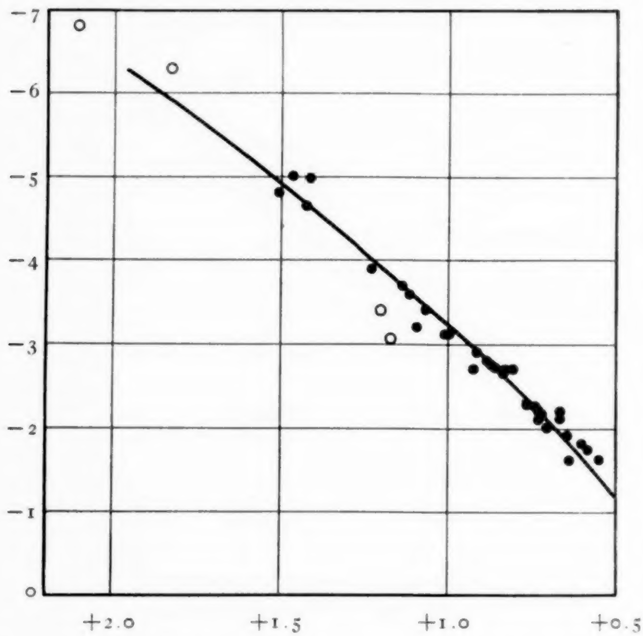


FIG. 1.—Period-luminosity curve. Ordinates are absolute visual magnitudes; abscissae are logarithms of the period.

alterations of the adopted quantities (5) are introduced. The value of m adopted by Eddington for theoretical work is 2.8. In an unpublished investigation Russell uses a value very slightly smaller. Jeans, Eddington, Lindemann, and others have noted that m must be of the order of magnitude here used on account of the necessarily high degree of ionization that is believed to prevail at stellar temperatures.

[1] *Mt. Wilson Contr.*, No. 154, 1917; *Astrophysical Journal*, **49**, 24, 1919.

Accepting, therefore, the radiative theory of giant stars and a most probable value of the average molecular weight, we conclude from the foregoing discussion that for all typical Cepheid variables μ/R has practically the same value;[1] and we may reasonably infer that this condition of definite average heat content and corresponding critical central temperature is associated with the origin, and particularly with the persistence, of the pulsations that are believed to underlie the spectral, light, and velocity variations.

TABLE IV

A Theoretical Period-Luminosity Curve

β	μ	M	log P
0.2	70	−6.27	1.97
0.3	29	−5.17	1.59
0.4	15	−4.28	1.31
0.5	9	−3.53	1.08
0.6	5.5	−2.76	0.87
0.67	4.0	−2.20	0.73
0.7	3.5	−1.95	0.67
0.8	2.2	−1.01	0.47

The duration of Cepheid variation is much the same problem as the duration of the giant stages of all gaseous stars. Both problems involve principles of radiation or sources of energy beyond those currently accepted. Russell has made the interesting suggestion that the maintenance of the pulsations may be ascribed to the pulsatory supply of heat from some undescribed internal source at times of greatest contraction.[2] Without some such supplementary means of sustenance it appears improbable that the pulsations could long continue. It seems likely, therefore, that only those pulsatory disturbances, starting with the critical μ/R (and the resulting condition of internal temperature) that is favorable for drawing on the supplementary sources, would sufficiently develop and persist to result in typical stellar variation.

[1] Eddington's observation (*Monthly Notices*, **77**, 2, 1918), that for galactic Cepheids the central temperature divided by β is essentially constant, is equivalent to this result, for on his theory of radiative equilibrium the central temperature is proportional to $\mu^{\frac{2}{3}} \bar{\rho}^{\frac{1}{3}} \beta$; cf. equation (1′) above.

[2] *Publications of the Astronomical Society of the Pacific*, **31**, 205, 1919.

V. NOTE ON THE DISTANT CLUSTER N.G.C. 7006

Observations of the extremely distant globular cluster N.G.C. 7006 have been discussed in preceding papers of this series.[1] In addition to the extremely small parallax, the study thus far has given results that bear on the scattering of light in space, on the comparability of near and distant clusters, and on the speed of evolution of giant stars.

A photograph made with the 60-inch reflector in July, 1918, by Professor J. C. Duncan, affords additional data of interest in the comparison with the brighter systems. The exposure of 95 minutes on a Seed 27 plate shows the cluster as a typical system, differing very little in general appearance from Messier 15. A study of the photograph yields the following results:

1. The brightest ten stars of the cluster are photographically somewhat fainter than the seventeenth magnitude. The faintest on the plate are of the twentieth magnitude, corresponding, at the adopted distance of 67,000 parsecs, to the absolute photographic magnitude +1 and to a luminosity one hundred times as great as that of the sun.

2. In the field of the cluster the foreground density averages 6.4 stars per square minute; that is, to the twentieth photographic magnitude there are in this part of the sky about 23,000 stars in a square degree. The galactic longitude and latitude are 32° and −20°. Seven faint nebulae, apparently spirals, are on this plate within ten minutes of arc of the center.

3. Two counts with a superposed réseau give a mean of 505 cluster stars, 200 of which are brighter than magnitude 18.5. The numbers agree very closely with the numbers of stars in Messier 3 and Messier 13 for the corresponding intervals of absolute magnitude, thus emphasizing further the similarity of near and distant globular clusters.

4. The cluster appears to be slightly elliptical. Eight independent estimates give values of the position angle of the major axis ranging from 90° to 135°, with a mean of 115°. The star counts, grouped in quadrants, afford a rough verification. On the Franklin-Adams chart the cluster appears as a diffuse star image

[1] *Mt. Wilson Contr.*, No. 152, p. 14, n. 1; *Astrophysical Journal*, **48**, 154, 1918; No. 156, pp. 1–6; *Astrophysical Journal*, **49**, 249, 1919.

and no elongation can be detected. New measures of the diameter on the chart give 46″, in nearly exact agreement with earlier determinations. A distinct (but probably chance) spiral structure can be traced on the plates of shorter exposure; it is absent, however, from this new plate, which brings out the fainter stars.

5. The angular diameter of the cluster on the Mount Wilson photograph is approximately four minutes of arc, corresponding to a linear diameter of 80 parsecs. Extension to still fainter magnitudes would probably give a slightly larger diameter.

Mount Wilson Observatory
May 1920

PHOTOGRAPHIC PHOTOMETRY AND THE PURKINJE EFFECT[1]

By FRANK E. ROSS

ABSTRACT

Contrast properties of photographic plates.—The variation with wave-length of the contrast or gamma and of the rate of increase of the diameters of stellar images with the exposure, which the author calls *astrogamma* (Γ), is shown for four emulsions by curves (Figs. 1–4). These results, which lead to a photographic Purkinje effect, are explained as due in part to *the variation in the penetration of light of different wave-lengths*, which the author determined from sections of images (Plate VI), and in part to variations in the number of silver grains sensitive to various colors. Other factors also enter.

Heterochromatic photographic photometry.—After discussing the criteria to be satisfied by an accurate photometric method, the author points out the *possibility of large error* unless γ is constant for the range of wave-lengths involved or unless filters specially adapted to the particular plates are used. From the curves given, it is evident that orthochromatic and panchromatic plates are more suitable for photometric purposes than ordinary blue-sensitive plates. In applying photographic photometry to astronomical problems, extreme circumspection is necessary to avoid erroneous conclusions.

One of the most important applications of photography to astronomy is in the field of photometry, the measurement of intensity of radiation. Visually two intensities are considered to be equal when they produce equal visual sensations. Photographically they are considered equal when they produce equal blackening, or, in the case of stars, when they produce disks of equal size. Astronomers have been not without concern as to the truth of these postulates, certain aspects of which are to be critically examined in the present paper.

What are the criteria which must be applied to any given photometric method in order that it may be a true method? These criteria appear to be two in number, an intensity and a time criterion. Before considering them in detail it will be convenient to define a term, "photographic sensation." Two intensities are considered to produce equal photographic sensations when they produce equal blackening or density in the same time, or, in the case of stars, when they produce disks of equal size. The intensity

[1] Communication No. 95 from the Research Laboratory of the Eastman Kodak Company.

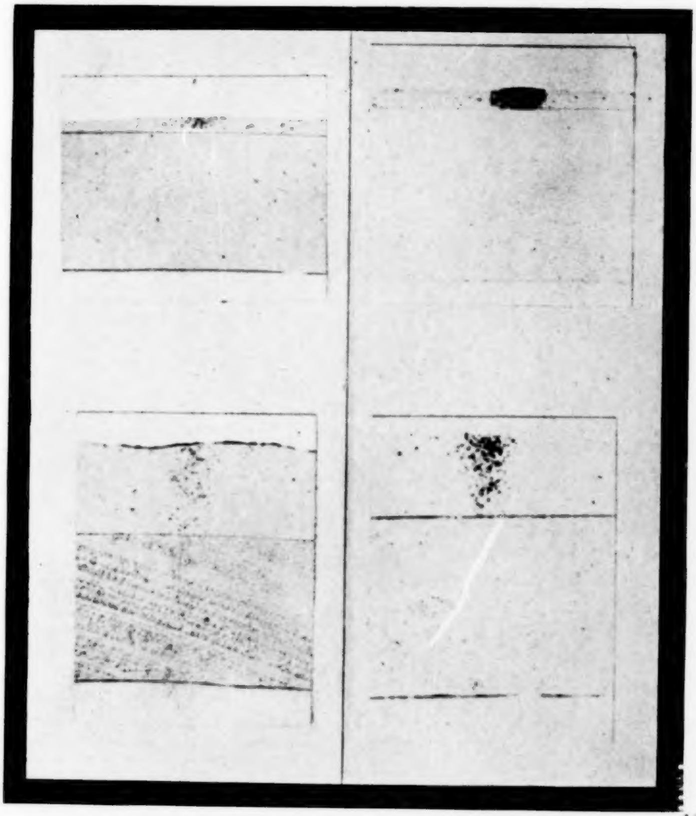
PLATE V

Diameter = 0.050 mm

Blue Yellow

Dry

Wet



MAGNIFIED SECTIONS OF STELLAR IMAGES

criterion requires that if two sources produce the same photographic sensation, and are accordingly judged to be equal, the photographic sensations must remain equal when both sources are increased or decreased in the same ratio. The second criterion is the same as the first with changes in exposure-time substituted for changes in intensity. It can be affirmed that these criteria are satisfied provided the two radiations falling on the plate are of the same quality, and provided they fall on the plate in exactly similar manner as to direction and distribution. In any practical problem, however, such complete equivalence cannot be maintained. It is at this point therefore that photographic-photometric investigations must start if there is to be any certainty in the accuracy of the results obtained. To enumerate only a few of the problems arising in this connection, the effect of color or wave-length must be determined, since complete equivalence in quality of the illumination cannot always be maintained. The effect of lens aberrations of all kinds must be considered since the images to be compared necessarily lie in different portions of the field of the optical system. The effect of stopping down the objective, producing changes in diffraction and aberration patterns, and changes in the direction of incidence of the light on the plate, is to be considered. These effects must be investigated separately for intensity and time-variations and moreover for the various classes of photographic plates. It is quite conceivable that deep or shallow development may cause variation in the relative actions, so that the effect of developers should be investigated as well.

Only one of the foregoing effects is to be considered in the present paper, the color or wave-length effect. As already pointed out, photographic sensations are of two distinct kinds, equivalence of density and equivalence of diameter. Any investigation of the effect of color on photographic sensation must consider these divisions separately, as it is quite conceivable that different laws might be obtained in the two cases.

Two distinct properties of the photographic plate are involved in its use for photometric purposes, namely:

a) *Threshold sensitivity.*—This is of great practical importance, but does not concern us in the present problem.

b) *Contrast.*—The properties of the contrast of a photographic plate are of exceedingly great importance in photographic photometry. They are comparatively little known or understood. In photographic literature the contrast of a plate is called gamma

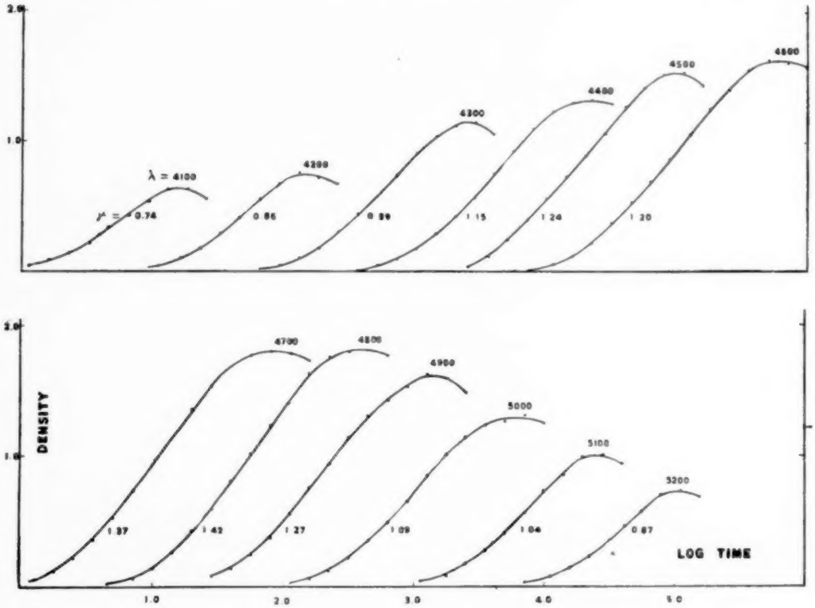


FIG. 1.—Characteristic curves of a fast blue-sensitive plate

(γ) and is defined as the rate of growth of density D with logarithmic increase of exposure ($E=I\,t$), i.e.,

$$\frac{dD}{d \log E}=\gamma\,.$$

By analogy the rate of increase in the diameter Δ of photographic stellar images will be called astrogamma (Γ), i.e.,

$$\frac{d\Delta}{d \log E}=\Gamma\,.$$

In general the properties of γ and Γ are similar. Gamma depends upon developer and degree of development. Astrogamma, how-

ever, is influenced by these factors only to a very small extent. Both depend upon the emulsion and upon the wave-length of the

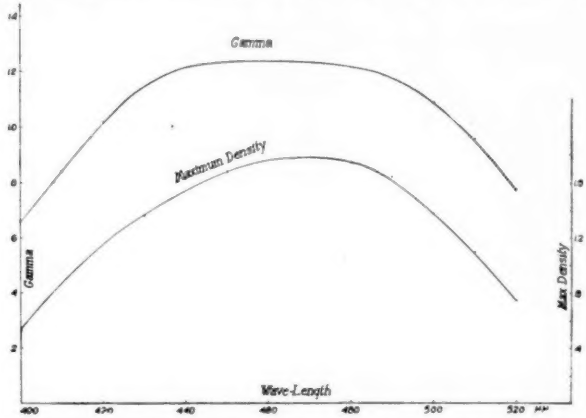


FIG. 2.—Curves for gamma and maximum density derived from Figure 1

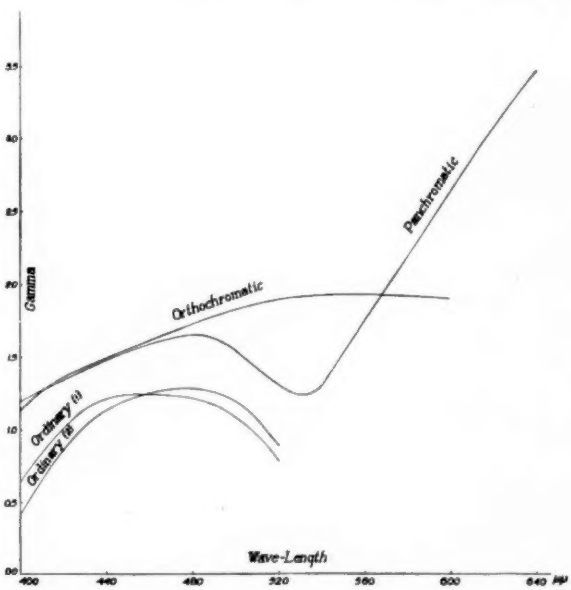


FIG. 3.—Variation of gamma with wave-length for various plates

incident light. Figure 1 shows the extent to which gamma depends upon wave-length. It gives the characteristic curves of a fast

emulsion exposed to monochromatic light of wave-length 4100 to 5200 A. (A characteristic curve is a graph of densities plotted as ordinates against logarithmic exposures as abscissae.) It is seen that both gamma and maximum-density vary considerably with wave-length. These values are plotted in Figure 2.

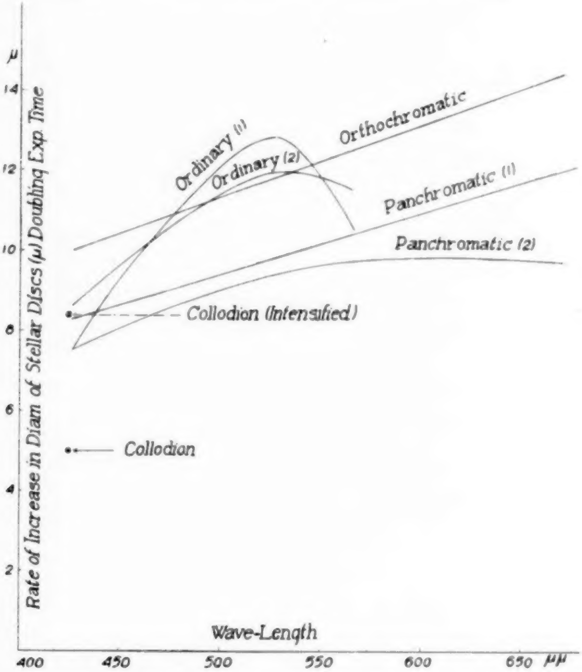


FIG. 4.—Variation of astrogamma with wave-length for various plates

In Figure 3 gamma wave-length curves for four emulsions are given, a panchromatic emulsion, an orthochromatic and two ordinary emulsions. In Figure 4 the corresponding astrogamma wave-length curves are shown.

Consider briefly the effect of variations in gamma or astrogamma with wave-length upon photographic photometry. Imagine two stars, a blue star S_b and a red star S_r, photographed on a panchromatic emulsion, and let the photographic sensation for S_b be greater than for S_r, with a given exposure-time. Referring to Figures 3 and 4 it will be seen that if the exposure-time is increased, the photographic sensation for S_r will gain on that for S_b, at a

certain point becoming equal to it, and then surpassing it. A similar result is found if intensity-variation is substituted for time-variation. If the method is a true photometric method, a blue star k magnitudes greater than S_b should produce a photographic sensation equal to that for a red star k magnitudes greater than S_r, provided the sensations for S_b and S_r are equal. Again the curves show that this is not necessarily true and that the star KS_r may exceed the star KS_b in photographic sensation. The conclusion is that the photographic plate in common with the eye is incapable of heterochromatic photometry, in the strict sense of the term. This is of course well known. The point of interest and uncertainty is the magnitude of the effect in any given case.

According to Hertzsprung[1] the effective wave-length of the average A-type star is 4266 A, with a change of 200 A for a change of unity in color-index. Consider two stars whose difference in color is represented by unity on the astronomical scale, one an A-type star, the other a K-type star, and suppose the out-of-focus photometric method is used, with "blackness" as the criterion for magnitude. From the gamma wave-length curve for ordinary emulsions (Fig. 2), the gammas are respectively 1.11 and 1.23 for the two stars considered. Let the photographic sensations or "blackness" be equal for S_a and S_k for an exposure-time t_0. Since

$$D=D_0+\gamma \log t ,$$

if the exposure-time is increased tenfold, it is clear that the density for the K-type star will now exceed that of the *A*-type by 0.12. Assuming the reciprocity law, the equation for the magnitude M is

$$D=D_0+\frac{2}{5}\gamma M ,$$

from which

$$\delta M=2.14\,\delta D .$$

Since $\delta D=0.12$, $\delta M=0.26$, which is the amount the magnitude of the K-type star now nominally exceeds that of the A-type. In other words, there is an uncertainty of 0.26 in the relative

[1] *Astrophysical Journal*, **42**, 92, 1915.

magnitude of the two stars, depending on the particular exposure-time chosen. In general the difference is a linear function of the difference in the logarithms of the exposure-times and of the gammas corresponding to the effective wave-lengths. It can be easily shown to be

$$\delta M = 5\delta \log t \, \frac{\gamma_B - \gamma_A}{\gamma_B + \gamma_A},$$

where A and B refer to the two stars.

It is to be pointed out that these computations are correct only in case the reciprocity law holds. If Schwarzschild's law holds, the effect is increased in the ratio $1:p$. There is considerable evidence that neither the reciprocity nor Schwarzschild's law represent the correct time-intensity action on the photographic plate. (For a discussion the reader is referred to a paper by the writer in the *Journal of the Optical Society of America*, September 1920.) So long as the reciprocity law is assumed to hold, photographic photometry takes on a simple form. It becomes only slightly more complex under Schwarzschild's form of the law. If, however, Kron's law or some more complicated law holds, photographic photometry becomes increasingly difficult and enveloped in uncertainty. The series of characteristic curves for various wave-lengths shown in Figure 1 were made on a time-scale (intensity constant, time variable). If they had been made on an intensity-scale (time constant, intensity variable) no difference would be found provided the reciprocity law holds. If Schwarzschild's law is the correct one, the only change in the curves would be a magnification of the ordinates in the ratio $1:p$, there being no change in the relative values for various wave-lengths. But if neither of these laws hold, this would not be true. In that case the gamma wave-length curve and the maximum-density wave-length curve (Fig. 2) would depend upon the ratio of the light-intensity acting to the sensitivity of the plate for the particular wave-length. Accordingly any mathematical formula expressing the photographic effect in terms of time and intensity must take into consideration the spectral characteristics or quality of the light as well as the spectral sensitivity of the plate.

There is evidence indicating that the low gamma and low maximum-density at λ 4100 and λ 5200 shown in Figure 1 are due partly to low light-intensity and low sensitivity, or more strictly to a low ratio of the two. A corollary of great importance is that light of very feeble intensity produces a low photographic density which cannot be increased by increasing the exposure-time. This is in agreement with H. S. Channon's[1] experimental results. Channon attributes the phenomenon to the reversal action.

If the effective wave-lengths of the stars all lie on the horizontal portion of the gamma wave-length curve, no ambiguity such as that which has just been calculated can arise. The effect is in general due to the violet and ultra-violet light, and can obviously be decreased by using a suitable yellow filter. It would seem that in common with the green and blue-green portions of the spectrum violet and ultra-violet radiation produce comparatively low contrasts or gammas and low maximum-densities in the majority of photographic emulsions, a matter of vital concern in accurate photometric work. Gamma wave-length curves for all emulsions useful in astronomical photometry should be measured and filters matched with each in such a way as to make the effective radiations produce approximately equal gammas. It will be noted in Figure 3 that the gamma wave-length curve of the orthochromatic emulsion is without the dip in the blue-green and is accordingly very valuable for photometric work. This result, however, remains to be checked. If emulsions can be found which give curves of this type they should prove exceedingly useful in photometric work. In any case Figures 3 and 4 conclusively show that orthochromatic and panchromatic plates are more suitable for accurate photometry than ordinary or blue-sensitive plates. Note the greater steepness of the curves throughout the violet in the case of the ordinary plates, a characteristic which impairs their usefulness for photometric work in a vitally important region of the spectrum.

That there is an enormous difference in the photographic action in star images for light of different colors is apparent from an examination of the images themselves. To show this the writer

[1] "Studies in Photographic Science," *Photographic Journal*, **50**, 164.

exposed an artificial star in a precision camera with blue and yellow filters interposed in turn. Orthochromatic film was used. For the moderately small diameter of image of 0.050 mm the image of the blue star was exceedingly weak and gray, showing that it lay in a thin layer near the surface, while the image of the yellow star of the same diameter was very black. Sections were made of these star images which are shown in Plate V. The difference in penetration is to be noted.

To show further the effect of color, sections were made of sensitometric strips which were obtained by exposure to monochromatic light of wave-lengths 4300 and 5600 respectively. As before, orthochromatic film was used. In order to make the results comparable, equal densities were chosen. The increase in penetration of the image for the longer wave-length is apparent (Plate VI).

In order to understand the actions here discussed and the reason for the change of gamma with wave-length, it is necessary to gain some idea of what contrast or gamma really is, and to apprehend its relation to some of the other physical constants of the emulsion. Consider two normal exposures: E, developing to a density D_1; $10E$, developing to a density D_2. In this case by definition

$$\gamma = D_2 - D_1 .$$

Before proceeding farther it is necessary to define the photometric constant c. This is defined as the weight of silver in a square centimeter of a developed emulsion whose density is unity (transmitting one-tenth of the incident light). The photometric constant has been found by experiment to be 0.100 milligrams. It has also been found experimentally that the weight of silver is proportional to the measured density, unless the density is very high, so that the equation

$$M = c \cdot D$$

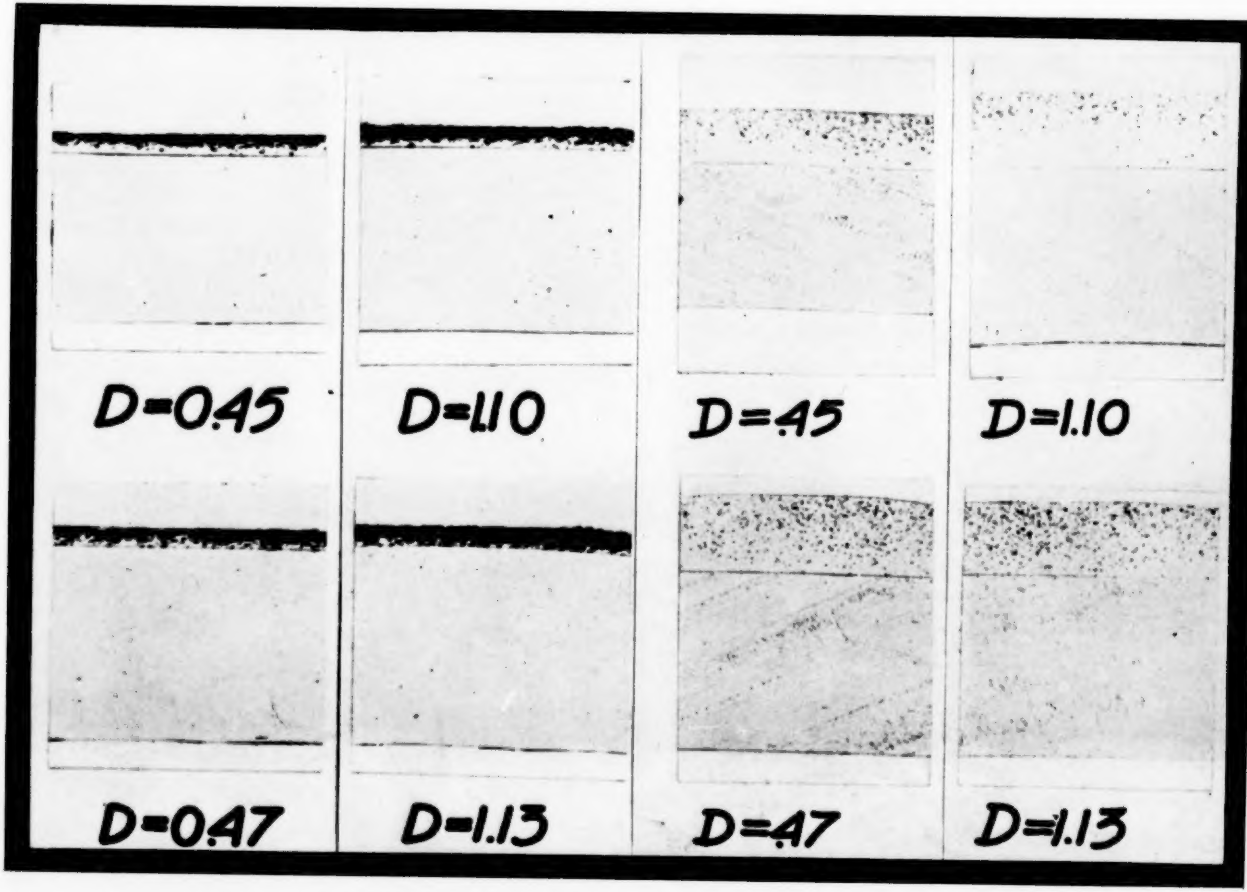
holds, where M is mass of reduced silver.

Consider now the wave-length changed to a value giving double the penetration into the film, other conditions remaining unchanged.

PLATE VI

4300 A

5600 A



Magnified Sections of Densitometric Strips at Two Wave-Lengths

Magnified Sections (Wet)

It is evident from the mass-equation above that the densities D_1 and D_2 will be doubled, so that the new γ will be

$$\gamma_1 = 2D_2 - 2D_1 = 2\gamma\ ,$$

or γ is also doubled. It is clear then that gamma is directly proportional to the penetration of the radiation into the film, other things being equal. The absorption and scattering of a silver-bromide emulsion in the ultra-violet is exceedingly strong, leading to low penetration and so to a low gamma. In photographs of the spectrum the low value of gamma in the ultra-violet is very noticeable when comparing negatives made with short and long exposures. With increasing wave-length the scattering and absorption are less. The radiation accordingly penetrates to deeper layers of the emulsion, leading to increasing values of gamma.

It is evident, however, from an inspection of Figure 3 that penetration is not the only factor governing gamma. The dip in the curve in the green portion of the spectrum cannot be explained as being due to less penetration. The sensitivity-curves show similar depressions at the same wave-lengths.

It has been shown[1] that the characteristic curves of emulsions probably result from a certain normal action. Let there be M silver grains per unit volume in the emulsion. Let there be N grains per unit volume participating in the normal action. For a thin layer the normal action is

$$dn = \kappa I(N-n)dt,$$

where dn is the number of grains changed to latent image in the time dt; n the number changed up to the time t; κ a constant for a particular plate and wave-length; I is light-intensity. The reversal action is neglected.

Many phenomena indicate that N is less than M, i.e., all the grains in an emulsion do not participate in the normal action. Consider for the moment a photo-electronic theory of the latent image. Imagine with J. J. Thomson that the energy of the impinging radiation is in filaments, and in quanta as well, and therefore

[1] "On the Relation between Photographic Density, Light-Intensity, and Exposure-Time," *Journal of the Optical Society of America*, September 1920.

equivalent to a stream of material particles. Suppose that a collision of one quantum considered as an entity, or of a definite number of quanta, with one or a definite number of valency electrons in a silver halide grain is necessary and sufficient to make that grain developable. It follows at once from the laws of chance that a *constant fraction* κI of the grains uncollided with at any time t will suffer collisions in the next time interval dt and accordingly be made developable, which is exactly the law of normal action specified above. This presupposes that all the grains N are in the same condition, or of the same sensitivity. Since the light quanta contain the frequency as a factor, a wave-length effect should be present, that is, N is a function of the wave-length. The statement above that the grains should be of the same sensitivity must be qualified. For emulsions of different sensitivities may be mixed and still normal action result. It is more accurate to state that grains are normal when they can be placed in one of a limited number of groups according to sensitivity, the extreme range of sensitivities being not too great, less than one to a hundred, say. It is shown in the paper by the writer (*loc. cit.*) that gamma decreases as the number of groups increases, and that the highest value of gamma is obtained when all the grains are of the same sensitivity.

Briefly then we can consider that an emulsion contains a certain amount of inert silver grains varying with wave-length between certain limits, which is dissolved out in the fixing bath. It was shown above that gamma contains the mass of silver as a factor. Since the inert silver disappears it does not figure in the effective mass. It is clear then that gamma and maximum-density must decrease the greater the amount of inert silver in the emulsion. It is this effect, depending on wave-length, combined with the penetration effect already considered, which gives the resultant curve of wave-length gamma. The phenomenon of "reversal" will doubtless be found to play a part. In the present state of the science of photography it is impossible to estimate the relative importance of the various factors.

Development undoubtedly plays a part in determining the mass of the inert grains. On account of differences in the reduction-

potential of developers and differences in other properties as well, the latent image is not uniformly developable. This is clearly brought out by differences in maximum-gamma and maximum-density obtained with developers of various types.

It cannot be too strongly insisted that the true significance of measures and results obtained by photographic photometry in any particular case should be considered with reference to the particular problem being investigated. In so far as the physics of the heavenly bodies depends upon photometric measures and scales, extreme circumspection is necessary in order that correct conclusions may be reached. For example, in the case of faint stars requiring very long exposures, the shorter wave-lengths ($\lambda < 4400$) have a comparatively negligible effect upon the photographic plate (see Fig. 1). This is due to the low penetration of the feeble short waves, leading to a low maximum density as just shown. Given a faint star emitting short and long waves, the photographic effect of the short waves, as the exposure-time is increased, very soon reaches its peak and declines owing to reversal, while the effect due to the longer waves is still on the increase. Although the application of a simple addition-theorem to a case of this sort may be open to question, it seems certain that the proportional effects of the various wave-lengths are not constant for variations of time or intensity. It would appear to be true from these considerations that for faint stars the short wave-lengths are comparatively without effect, leading to a classification of faint stars as redder than they actually are.

I am indebted to William Herriott, of the Research Laboratory of the Eastman Kodak Company, for assistance in accumulating data and for the sections of star images and of densitometric strips.

Eastman Kodak Company
Rochester, N.Y.
May 17, 1920

IMAGE CONTRACTION AND DISTORTION ON PHOTOGRAPHIC PLATES[1]

By FRANK E. ROSS

ABSTRACT

Contraction and distortion of photographic images during drying.—When pyrometol or caustic hydroquinone are used, extended images contract by an amount depending on the developer, on the size and density of the image, and on the temperature. The contraction reaches a maximum for images 5 mm or more in diameter. The increasing contraction with increasing density of image may be sufficient to overcome the normal increase of size with density so that the diameter of the image may actually decrease to a minimum as the exposure is increased. The effect is especially marked in hot weather. The phenomenon is probably a result of unequal drying of image and surrounding gelatine. Hydroquinone, metol-hydroquinone, and chlorhydroquinone seem to give images free from this contraction effect.

Astronomical measurements from photographs.—*The possible errors* due to the contraction effect may be large. Images of *double stars* may be too close together; star images near the solar corona may be attracted so as to lead to apparent deflections which must be corrected for if the *Einstein effect* is to be determined.

Attraction of close lines on spectrograms.—This contraction effect depends on the developer and on the width and separation of the lines.

The condition of orthoscopy in photographic reproduction depends upon two factors: (*a*) the optical system; (*b*) the photographic plate or emulsion. Only distortions of the latter kind will be considered.

In studying the question of the extent to which reproduction of dimensions can be obtained with the photographic plate, two cases are to be distinguished:

a) Size of images on the plate small in themselves and small with respect to the distances separating them. This is the case of most interest in astronomy and one that has been studied by numerous investigators.

b) Size of images large, with no limitation on separating distances. Investigation of this phase divides itself into two parts: (1) study of dimensional changes in the photographic images; (2) study of relative displacements of image centers.

[1] Communication No. 94 from the Research Laboratory of the Eastman Kodak Company.

Case (*b*) will be considered first, as it is one of greater generality and, as will be seen, involves the phenomena governing the action to their full extent.

In astronomical photometry, as is well known, there is a linear relation between stellar magnitude and diameter of image over a long range of magnitudes. This relation persists even to the threshold diameter, at least under laboratory conditions. There appear to be but two phases: (*a*) the induction period, during which no photographic effect is observable; (*b*) the nearly sudden appearance of an image, composed of but a few grains, which increases in size, without apparently pausing to gain density, at a uniform rate, and moreover without changing its initial or threshold rate of increase. This is in marked contrast to the condition obtaining in the more commonly used method of photographic photometry in which light-intensities are measured by *density* of silver deposit. In this case there are four periods: (*a*) the induction period, no deposit being visible; (*b*) the post-induction period, in which the increase of density, after the first appearance of the image, is at a rate so low as to be useless for photometry. This portion of the density-exposure curve is usually called the "toe"; (*c*) a period of rapid change of density in a linear manner with respect to the logarithm of the exposure. This is the only useful portion of the curve photometrically and is usually designated the straight-line portion. Its inclination to the log exposure axis is called gamma (γ); (*d*) a period of diminishing change of density, finally becoming zero. The name "shoulder" is given to this section of the curve. In astronomical photometry, on account of its greater accuracy, the photometric method depending on measurements of density is considerably used. It is of limited range, however, compared with the method based on stellar diameters, being useful over a range of only 5 or 6 magnitudes as compared with a range of about 10 magnitudes for the latter method.[1]

Departing from consideration of the point source, Figure 1 shows the relation between exposure-time and image-diameter for various sizes of image obtained from several series of accurate

[1] These ranges, which are based on laboratory measures, are greater than found in astronomical practice, which are respectively 3 to 4 and 5 to 6 magnitudes.

measurements. A new phenomenon is apparent. The image does not appear to increase its dimensions from the threshold point, the difference in action being seen to depend upon the size of the threshold or geometrical image. Following the induction period there is a lag and even a drop in the diameter, which increases in amount until it reaches a maximum. The diameter of the threshold image is roughly five millimeters at the point of maximum lag.

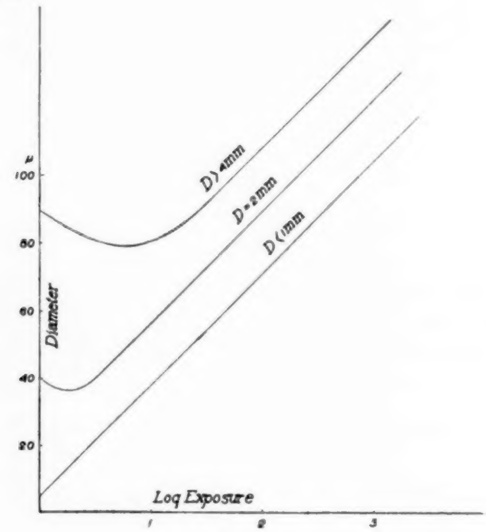


FIG. 1

The normal growth of an image with exposure is thus seen to be subject to interference by an amount depending on the size of the threshold or geometrical image. It may be inferred that there is a contracting influence making itself felt. A study of this phenomenon is the object of the present paper.

As early as 1915, while studying the turbidity of emulsions by measurement of the width of a slit printed on the plate by contact exposures, it was noticed that the width of the slit images for low exposures was always less than that of the slit itself, and, moreover, that there was no increase in width such as is always obtained with star images, until a high image-density is reached. The cause remained unknown until July, 1918, when it was definitely proved

from experiments that contraction in images occurs of an amount depending on a variety of conditions. It was shown that the contraction occurred during the drying operation, and that the immediate physical cause was one and the same with that producing general distortions of the photographic film, which has been the subject of so much investigation.

The test object shown to scale in Figure 2 was made for the purpose of investigating the subject of the contraction of an image

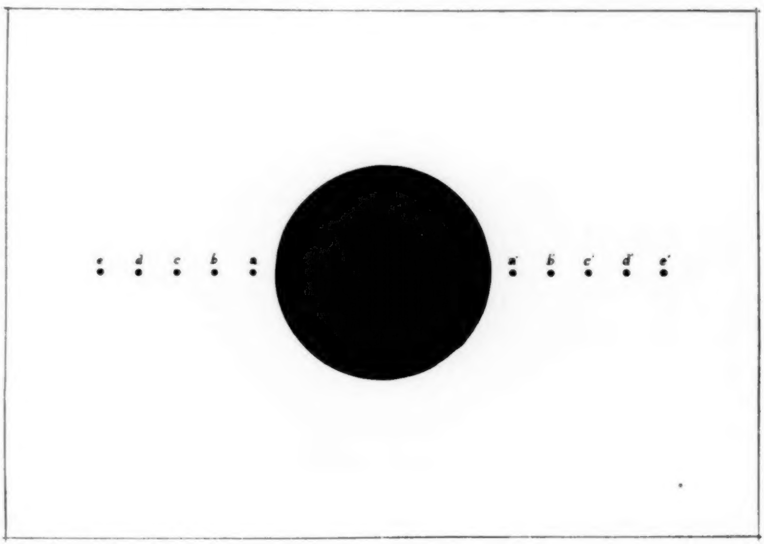


FIG. 2

and its possible effect on the position of smaller or point images lying at varying distances from it. It consists of a large circular hole 35 mm in diameter cut from cardboard and surrounded in the manner shown by smaller holes 1 mm in diameter. A plug for the large hole was provided, so that exposures could be made with or without the large central image. Clearly this will enable the influence of the central image on the position of images lying at varying distances to be studied. Two series of exposures were made on separate plates. These plates are one inch wide and four inches long. The camera is the one used in tests of resolving power, reducing 20 diameters. It is of great rigidity and stability,

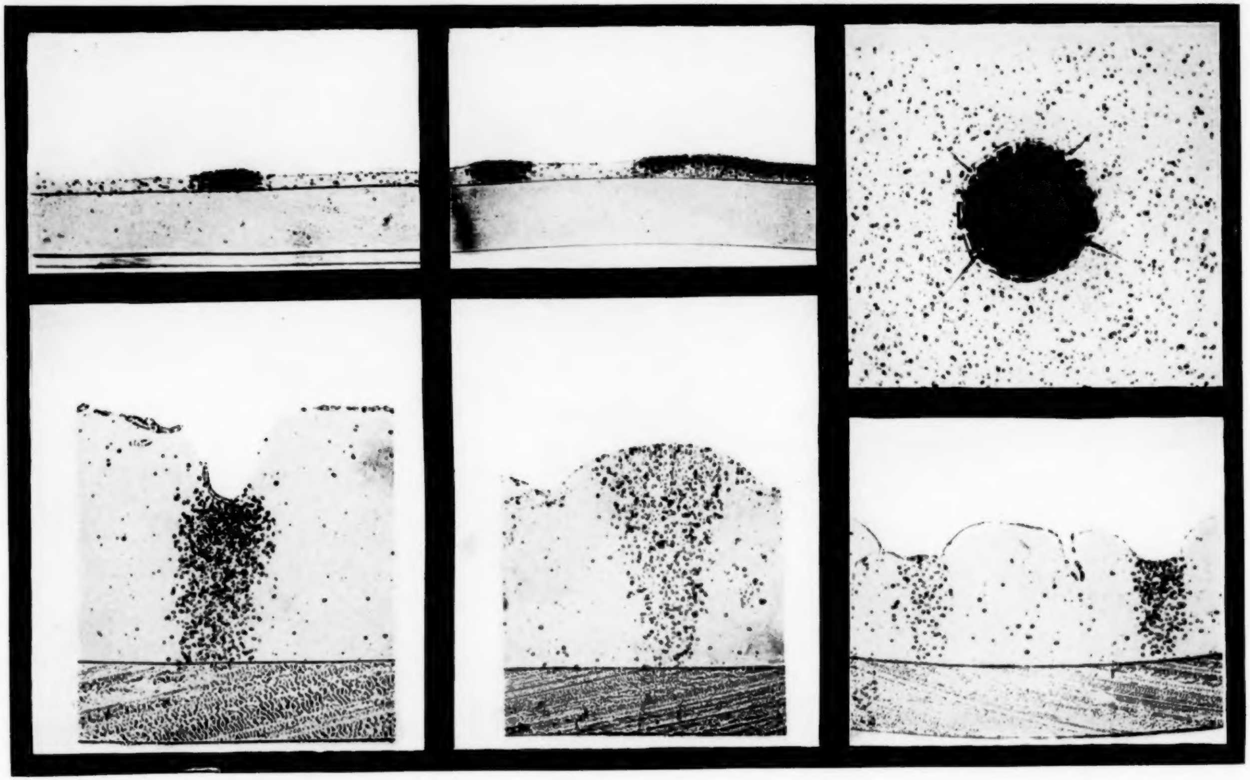
lens, plate, and object being firmly fixed to a massive iron tube. Ten exposures were made on each plate in two groups, exposure-times being in all cases 4^s, 6^s, 8^s, 16^s, and 32^s. The first group of five exposures was made with the central hole covered with its plug; the second group with plug removed. One plate was developed in a pyro-metol developer, popular in development of aerial negatives on account of the stain produced; the other was developed in a chlorhydroquinone developer. Fixing bath was without hardener. Each plate was measured twice, first while in the wet condition, and again after drying. When an emulsion is wet, it is necessary to measure through the back in order to eliminate errors due to capillarity. In the table below, which gives the result of the measures, *aa′* designates the distance between the pairs of images *aa′*, etc. Only the mean values are given, so that each quantity is the average of five measured quantities and has an uncertainty of not more than 0.001 mm. All measurements were made on a Hilger comparator.

TABLE I

	Diameter of Disk mm	*aa′* mm	*bb′* mm	*cc′* mm	*dd′* mm	*ee′* mm	
			Plate I. Developer, Pyro-Metol				
Disk out	....	2.078	2.684	3.362	3.981	4.628	Measured wet
Disk in	1.747	2.074	2.685	3.361	3.984	4.631	
Disk out	....	2.080	2.685	3.361	3.984	4.629	Measured dry
Disk in	1.638	2.033	2.673	3.357	3.979	4.625	
Disk out	....	− .002	− .001	+ .001	− .003	− .001	Contraction in drying
Disk in	+ .109	+ .041	+ .012	+ .004	+ .005	+ .006	
			Plate II. Developer, Chlorhydroquinone				
Disk out	....	2.086	2.690	3.360	3.986	4.637	Measured wet
Disk in	1.732	2.077	2.682	3.360	3.982	4.630	
Disk out	....	2.081	2.688	3.363	3.985	4.636	Measured dry
Disk in	1.720	2.082	2.688	3.359	3.981	4.632	
Disk out	....	+ .005	+ .002	− .003	+ .001	+ .001	Contraction in drying
Disk in	+ .012	− .005	− .006	+ .001	+ .001	− .002	

PLATE VII

a *b* *c*



d *e* *f*

a, Pyro; *b*, Pyro-metol; *c*, Pyro-metol; *d*, Pyro; *e*, Hydroquinone; *f*, Pyro-metol

This table discloses several facts: (1) with the large central image absent it is seen to be immaterial whether the stellar distances are measured wet or dry, results from the two developers agreeing; (2) with the central image present, in the case of the plate developed in pyro-metol, there is a large contraction of the central image during drying, accompanied by a drawing together of the star images *a* and *a'* which lie near its edge, an effect which is seen to diminish rapidly as the distance from the edge of the central image is increased; (3) with chlorhydroquinone developer no certain effect of any kind is apparent.

To study this phenomenon of apparent shrinkage, sections were made of images developed severally in pyro and hydroquinone, which are shown in Plate VII *d* and *e*. The photomicrograms were made with the sections swelled with water. They show a seriously disturbed condition for pyro development, the image lying below the surface of the surrounding gelatine, while for hydroquinone development the image is even slightly above the surface. As further evidence of strong physical effects depending on development Plate VII *f* showing a double star is given. There is seen to be a fold in the surface of the gelatine between the two components. These disturbances are not peculiar to the wet condition. Plate VII *c* shows an enlargement of a star image, developed in pyro-metol, made when dry. Raylike lines are seen to radiate from the edge of the disk. Furthermore, the entire edge of the image is seen to be in a disturbed state. This is in line with results obtained in practice. For in measuring diameters of artificial star images of great sharpness and density, such as are produced when development is in caustic hydroquinone, it is found to be impossible to focus sharply on the edge, showing that disturbed optical conditions prevail similar to that shown in the photomicrogram. Plate VII *a* shows a section of a dry stellar image.

Effect of size on contraction.—A series of images with diameters ranging from 0.1 mm to 4.5 mm was measured for image contraction. The results are shown in the curve in Figure 3. The relation between contraction and image-size is seen to be exponential. It may be inferred from this curve that a maximum contraction is reached for an image-diameter of about 5 mm, remaining constant

for larger images. This suggests that the disturbance is localized in a band about 2.5 mm wide just within the edge of the image, the larger part being within 0.6 mm of the edge. That it is an edge effect is clearly indicated in Plate VII *b*, which shows a section of an extended image lying close to a star image. The disturbance here is in a band 0.14 mm wide, which appears to be the region of most violent disturbance. From Figure 3 the contraction is seen to be quite sensible for stellar images of sizes occurring in practice,

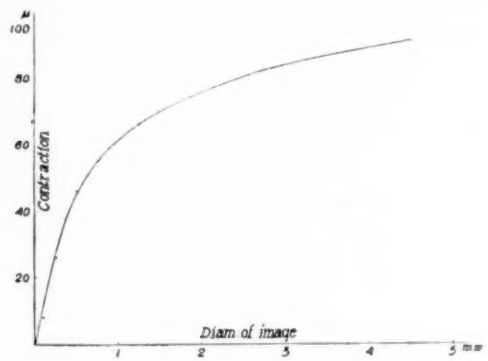


FIG. 3

amounting to 12 μ per 0.1 mm diameter. The normal growth of a star image under these circumstances is accordingly diminished 12 per cent through contraction.

Effect of density on contraction.—The contraction of a 4.5 mm image of various densities from medium to high was obtained in a similar manner. Results are plotted and the curve drawn in Figure 4. Here again an exponential relation is found between contraction and image-density.

The curves in Figure 1 can now be explained. For star images ($D < 0.5$ mm) the contraction is proportional strictly to the diameter, so that the only effect is a diminution of astrogamma.[1] Consider now the case of an image of 5 mm diameter, exposed to an increasing series of intensities. At threshold there is a normal *increase* in diameter, due to turbidity, which, however, is no

[1] Astrogamma is the rate of growth in diameter of star images with logarithmic exposure.

greater than for small star images. But as seen from Figure 1 the *contraction* is much greater than for small images, even at a low initial density. There is accordingly an initial *decrease* in size, which persists until the normal growth can make up the loss by contraction. The period of depression or of no growth is evidently a function of the size of image, with no change for diameters exceeding 5 mm.

It is clear that astrogamma depends to a certain extent upon those conditions which control image-contraction. In order to establish the nature of the dependence a series of plates was exposed to artificial stars and developed in many different devel-

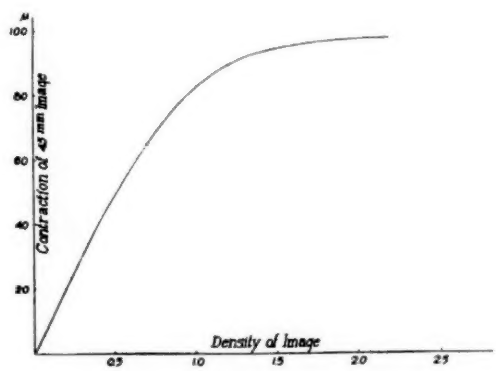


FIG. 4

opers. Astrogamma was determined for each plate both in the wet and dry condition. The wet plates were measured through the back, in order to eliminate refraction errors due to the capillarity present with sunken images, already mentioned. Details need not be given here. It will be sufficient to state that results were as expected, namely, that astrogamma decreased on drying, and that there was a variation with developer similar to that already found.

It remains to consider the cause of image-contraction, its wide range with developer, and possible dependence on other conditions. An indication of the immediate cause at least is to be found in the sections of star and of extended images already shown. In the

case of those developers giving greatest contraction, a strongly depressed wet image is found, while in the case of the developers giving little or no contraction a slightly elevated image is found. That particular physical property of gelatine which is most strongly affected by chemical action is hydration, or ability to absorb water. Sheppard and Elliott[1] have recently shown that one gram of gelatine absorbing 50 grams of water under certain conditions as to acidity will absorb only 10 grams when the acidity is only slightly changed.

Consider what takes place when a photographic emulsion dries on a plate. The natural tendency on dehydrating is to shrink equally in all directions, but on account of its strong physical affinity for glass it is unable to do so in a direction parallel to the plate. As a result there is a residual stress in this direction which is so strong that it may even cause the plate to take a slightly concave form. When an emulsion is stripped from a plate it will swell in its own plane only 25 per cent on complete hydration. It is clear from this that its horizontal elasticity or ability to swell has not been entirely lost. It is also clear that the physical structure of the gelatine has been profoundly modified by the stresses to which it has been subjected while drying on its glass support.

The phenomenon of contraction of photographic images and distortions in general can now be explained. On account of the smaller water content of the tanned developed image as compared with that of the surrounding gelatine, the image dries more quickly. Stresses parallel to the plate, at the edge of the image, which for brevity may be called horizontal stresses, accordingly become unbalanced, since the normally active counterbalancing stresses acting outward from the edge of the image have not yet developed owing to the still wet condition of the gelatine in the region surrounding the image. There is therefore a movement inward of the outer ring of the image. The movement should be greatest at the extreme edge, diminishing gradually toward the center, a diminution which is due to the accumulating reinforcement of the counterbalancing stresses. It has already been shown that this movement entirely ceases at about 2.5 mm from the edge. At this

[1] *Jour. Ind. and Eng. Chem.*, **10**, No. 9, 727.

point there must be a complete balance. With the edge moving inward, the gelatine in its immediate neighborhood must be dragged along by an amount which appears to be an exponential function of the distance from the edge. This accounts for the translatory motion of star images already described, which has been found above to be large in the immediate vicinity of the edge, decreasing rapidly with increase of distance.

An interesting application is to close double stars. There is considerable observational evidence that photographic measures of separation of close doubles are less than those obtained visually. It appears quite likely that there is a true contraction in this case. The separate images are so close together that the intervening gelatine must be in nearly the same condition as to moisture content as the images themselves. So far as this phenomenon is concerned, therefore, the two images act as a unit, so that contraction must take place toward the mid-point of the separating space as a center. The distance between the two true centers must accordingly diminish. The same is true of close spectral lines, which ought to show the phenomenon also. It should be mentioned at this point that experiments were made to determine the effect of the form of an image on its contraction, with special reference to this matter of spectral lines. It was found that a long rectangular image 1.7 mm in width contracted in width by the same amount as a square image measuring 1.7 mm on a side. The contraction of spectral lines should accordingly be governed by their width alone.

The caustic hydroquinone developer frequently used by astronomers on account of the density and blackness of the silver deposits produced, gives strong contraction effects, and should be avoided in lines of work where extreme accuracy is required with no suspicion of systematic error. Its effect on clear vision of the edge of the image necessary in accurate photometric work has already been noted. This is true for pyro developer as well. Straight hydroquinone, metol-hydroquinone, and chlorhydroquinone seem to give results in the main free from errors of contraction. A complete or extended classification of developers with reference to their contraction effect has not yet been made.

General distortions.—As already remarked, the subject of general distortion of the photographic plate is one that has received a great deal of attention from astronomers. The results obtained have been conflicting. The phenomenon of image contraction just discussed is not without enlightenment to the subject of general distortions, which will accordingly be considered from this same standpoint.

If a photographic plate is unevenly dried[1] it is subject to unbalanced stresses of the same kind as those just considered, so that distortions may be produced without the intervention of photographic images. Small images such as stellar disks may be displaced in position as a consequence. The original drying of the emulsion by the maker appears to determine the zero-point, so to speak, of the phenomenon. In this original drying the structure of the gelatine with its myriad channels is given its final form, to which it can be made to return by uniform drying, no matter what distorted intermediate forms it may have taken due to abuses such as faulty drying. This can be explained as follows: The stresses and displacements arising through abuse of the plate have been within the elastic limit, so to speak, of the emulsion, that is, they have not been great enough to produce any alteration of cellular form such as was produced in the initial drying of the emulsion. All displacements or distortions must therefore disappear under uniform drying, since the elastic limit has not been exceeded. This accounts for the fact that distortion produced by faulty drying may be caused to disappear on rehydration and correct drying.[2]

In small plates distortions are quite common, in which case alcohol hydration is recommended. In large plates, as is well known, there is a band at the edge a centimeter or more in width in which large distortions occur. Unbalanced stresses of the kind treated above are the immediate cause.

[1] A drying phenomenon has been noted by Mr. William H. Herriott of this laboratory which, while probably noticed before, is of sufficient importance to mention in the present connection. If a plate is dried with drops of water adhering to the back, drying is retarded at the points opposite. The drying is thus uneven, leading to possible distortions. The remedy is obvious. The cause is undoubtedly refrigeration due to evaporation.

[2] See *Special Publication No. 27*, U.S. Coast and Geodetic Survey, p. 44.

The subject of local distortion can be touched upon but briefly. Consider a réseau line or a stellar disk at the drying point, surrounded by relatively wet gelatine. Unless the horizontal forces so-called are perfectly balanced, there must be lateral shift of the image on drying. Lack of balance of the forces can be produced by local non-homogeneity of the gelatine or by lack of perfect symmetry of the image itself. The existence of local distortions is, however, problematical.

In measurement of plates containing images of sun or moon, there is chance of serious error due to the causes we have been considering. Measures of the diameter of sun or moon will be in general too small, as just shown. Star images in their immediate neighborhood will be correspondingly displaced. In planning work of the highest precision, it is accordingly well to bear in mind that there is a distortion at the edge of an image of the same nature as that at the edge of a plate, the region within and without the edge of the image corresponding to the disturbed region adjacent to the edge of the plate being in a source of danger.

The subject finds further application to measurement of the Einstein deflection of light at the solar limb. In this case the solar corona occupies the position of disturbing factor. In the case of eclipse expeditions to tropical regions it is extremely difficult to predict or to estimate the probable magnitude of the disturbance. This is due to the high temperature and humidity to which the plates must be subjected. In fact, during an exceedingly hot period in 1918, in which the temperature at Rochester remained near 90°F. for two weeks, exceedingly large distortions were consistently obtained, in spite of care in controlling temperature conditions in development and subsequent treatment. It seems probable that the high temperature to which the dry plates were exposed over an extended period influenced the gelatine to such an extent as to weaken its resistance to distortional forces.

A possible effect on the relative positions of stars in dense clusters is to be noted. This is of course a matter of importance in the study of the development of such systems.

EASTMAN KODAK COMPANY
ROCHESTER, N.Y.
May 21, 1920

THE PERTURBATIONS OF THE ORBIT OF THE SPECTROSCOPIC BINARY 13 CETI

By J. S. PARASKÉVOPOULOS[1]

ABSTRACT

Triple system 13 Ceti.—This interesting system is a visual binary with a period of 6.88 years whose brighter component is a spectroscopic binary with a period of about 2 days. During the seasons 1906–1907, 1908, and 1912–1913, 39 plates taken with the Bruce spectrograph at Yerkes Observatory have enabled the mean *elements of the orbit of the spectroscopic binary* to be determined for each of the three periods. These clearly show *perturbations due to the fainter visual component.* The period is shorter near apastron than near periastron. The eccentricity and $a \sin i$ show only slight variations but the precessional motion of the line of apsides is well established. The results enable the *relative masses of the components of the visual binary* to be computed approximately and indicate that the mass of the fainter component is probably at least as great as that of the brighter one. The *sign of the inclination of the orbit of the visual binary* is shown to be positive.

The star 13 Ceti ($\alpha = 0^h 30^m$, $\delta = -4°9'$), discovered by Hough in 1887 as a visual binary, has been found to have one of the shortest known periods (6.88 years). The elements of its orbit, as calculated by Aitken,[2] are the following:

$P = 6.88$ years	$\omega = 66°.8$
$T = 1905.27$	$i = \pm 53°.45$
$e = 0.725$	$r = 38°.7$
$a = 0''.242$	Angles increasing

Magnitudes of the two components 5.6 and 6.4

The brighter component of this visual system was announced by Professor Frost[3] to have a binary character, showing a variable radial velocity with an approximate period of two days. Therefore 13 Ceti is a triple system of great interest. Although the distance between the visual components of this system is always too small to allow them to be recorded separately by the spectrograph,[4] the spectrograms obtained record only a single spectrum

[1] Of the National Observatory at Athens. Volunteer Research Assistant at Yerkes Observatory, 1919–1921.

[2] *Publications of the Lick Observatory*, **12**, 5–6.

[3] *Astrophysical Journal*, **25**, 60, 1907.

[4] The limiting distance for the Bruce spectrograph of the Yerkes Observatory in practice is between 2″ and 3″.

belonging to the brighter component. Neither the spectrum of the fainter visual component nor that of the spectroscopic companion is visible in the spectrograms. The spectrum of this component is of type Go, according to the Henry Draper Catalogue.[1]

Professor Philip Fox, using eighteen spectrograms taken during the years 1906 and 1907 at Yerkes Observatory, obtained by Schwarzschild's method the following elements for the orbit of the spectroscopic binary.[2]

$P = 2.0818$ days	$T =$ J.D. 2417484.482
$\omega = 223°.1$	$a \sin i = 981{,}460$ km
$e = 0.062$	$K = 34.35$ km/sec.
$\mu = 172°.9276$	$\gamma = +10.5$ km/sec.

Though we do not know the distribution of the masses in this triple system, the presence of the third body (faint companion of the visual system) must undoubtedly produce perturbations in the elliptic motion of the spectroscopic binary, since it moves around the center of mass of the visual binary in an ellipse of great eccentricity (0.725). The determination of these perturbations is the purpose of the present investigation.

According to Aitken's elements given above, the spectroscopic binary should reach the periastron, the apastron, and the nodes of the visual orbit at the following dates:

Periastron........	1905.27	....	1912.15
Nodes...........	1905.76	1911.98	1912.64
Apastron.........	1908.71	....	1915.59

Spectrograms.—Since the discovery of the binary character of the bright component of 13 Ceti, spectrograms of this star have been taken at Yerkes Observatory with the Bruce spectrograph (one prism) during the years 1906, 1907, 1908, 1912, and 1913.

The series of spectrograms of 1906 and 1907 had already been measured by Professor Fox, and the first elements of the orbit deduced by him, but it seemed indispensable to the author for the

[1] Harvard *Annals*, **91**, 47, 1918.

[2] *Astrophysical Journal*, **27**, 375, 1908.

sake of uniformity of measurement to remeasure all the plates personally. All these measurements have been made with the Hartmann spectrocomparator, which is very convenient for spectra of the type of 13 Ceti.[1] As fundamental spectrum I employed one of the best plates of this star which I carefully measured on the

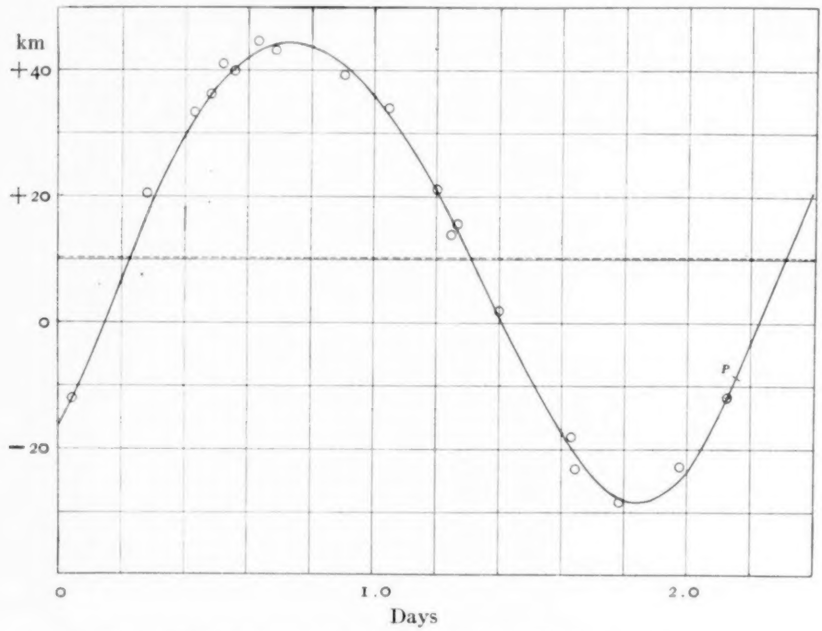


FIG. 1.—Velocity-curve of 13 Ceti from observations of 1906–1907

Gaertner measuring machine. The following lines of the stellar spectrum were used:

Mn 4035.883	Fe 4181.919	Fe 4260.640
Fe 4063.759	Fe 4195.492	Fe 4282.565
Fe 4071.908	Fe 4202.198	Ca 4302.692
Hδ 4101.900	Ca 4226.904	Fe 4325.939
Fe 4132.235	Fe 4233.328	Cr 4351.930
Fe 4149.533	Fe 4236.112	Fe 4404.927
		Fe 4415.293

Comparison spectra of titanium and of iron had been impressed on all the plates.

[1] Plates No. II B 87 and II B 90 (two-prism spectrograms) were not remeasured.

The portion of the spectrum in best focus was divided in nineteen regions, and in accordance with Hartmann's formula,[1]

$$M_2 = \frac{\frac{1}{2}(\Sigma d_1 + \Sigma d_2)}{\Sigma \frac{1}{s}},$$

the conversion factor s for each region corresponding to the wave-length of its center was computed. The different values of $\log \Sigma \frac{1}{s}$ corresponding to the various limiting regions employed in the measurements have also been tabulated.

STANDARD PLATE

I B 1909, 13 Ceti, 1908 December 21.530 (G.M.T.)

W = 7.7 (Standard Velocity = +34.25 km/sec.)

Region	λ	s	Region	λ	s	Region	λ	s
1......	4020	682	8.....	4235	822	15.....	4490	995
2......	4034	692	9.....	4270	845	16.....	4524	1019
3......	4062	709	10.....	4296	862	17.....	4546	1034
4......	4112	741	11.....	4335	889	18.....	4596	1069
5......	4138	758	12.....	4380	919	19.....	4641	1101
6......	4168	777	13.....	4406	937			
7......	4197	796	14.....	4452	969			

$\log \Sigma \frac{1}{s}$

Region	14	15	16	17	18	19
1................	8.24087	8.26524	8.28779	8.30892	8.32842	8.34655
2................	8.20266	8.22920	8.25366	8.27647	8.29744	8.31689
3................	8.16132	8.19043	8.21710	8.24186	8.26453	8.28547
4................	8.11685	8.14899	8.17824	8.20524	8.22985	8.25248
5................	8.06957	8.10525	8.13749	8.16706	8.19387	8.21840
6................	8.01776	8.05776	8.09357	8.12618	8.15554	8.18227
7................	7.96050	8.00586	8.04601	8.08224	8.11461	8.14390
8................	7.89624	7.94841	7.99396	8.03458	8.07056	8.10286
9................	7.82330	7.88437	7.93673	7.98279	8.02312	8.05899
10................	7.73826	7.81149	7.87277	7.92570	7.97141	8.01162

In the column headed "Observer" (Table I), F = Frost, B = Barrett. Mr. Frank R. Sullivan, engineer in charge of the telescope, assisted in securing the plates. Under n are given the number of regions measured for each plate. The usual time required for a satisfactory exposure was about two hours.

[1] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 53, p. 31.

TABLE I

Plate	Date (G.M.T.)	Julian Day	Exposure	Observer	V	n	V_C	O−C
	Observations 1906–1907							
		2,417	m		km		km	km
I B 872	1906 Oct. 1.787	485.787	75	B	+15.48	12	+14.22	+1.26
912	Nov. 9.589	524.589	105	B	+40.76	12	+38.08	+2.68
II B 87	Nov. 23.544	538.544	184	F-B	−22.68	12	−24.75	+2.07
90	Nov. 24.553	539.553	201	F-Fox	+39.03	12	+40.21	−1.18
I B 917	Nov. 27.514	542.514	101	F-B	−28.21	12	−27.82	−0.39
919	Dec. 1.542	546.542	92	F-Fox	−23.24	12	−21.13	−2.11
921	Dec. 3.610	548.610	92	F-Fox	−17.96	12	−20.13	+2.17
951	1907 Jan. 25.531	601.531	90	B	+33.22	12	+32.04	+1.18
1164	Sept. 13.872	832.872	130	F	+43.27	11	+44.00	−0.73
1170	Sept. 21.787	840.787	133	F-B	+20.56	12	+16.70	+3.86
1215	Oct. 20.696	869.696	120	F	−11.98	12	−12.03	+0.05
1244	Nov. 23.592	903.592	100	Lee	+44.49	12	+42.94	+1.55
1249	Nov. 25.601	905.601	120	Lee	+39.84	10	+40.30	−0.46
1255	Nov. 27.610	907.610	82	Fox-B	+36.17	12	+36.12	+0.05
1259	Nov. 30.604	910.604	120	Fox	+ 2.03	12	+ 1.54	+0.49
1264	Dec. 4.571	914.571	180	Fox	+21.15	12	+20.35	+0.80
1273	Dec. 6.503	916.503	112	Fox-F	+33.93	12	+32.20	+1.73
1277	Dec. 6.701	916.701	140	Fox	+14.00	10	+16.00	−2.00
	Mean	735.900						

	Observations 1908								
			2,418	m		km		km	km
I B² 1731	1908	Sept. 8.825	193.825	102	Lee-B	− 8.38	6	− 2.64	−5.74
1808		Oct. 30.699	245.699	60	B	+17.17	9	+12.26	+4.91
1833		Nov. 8.608	254.608	120	F	−29.88	12	−28.87	−1.01
1852		Nov. 13.614	259.614	69	B	+39.71	12	+41.84	−2.13
1859		Nov. 16.500	262.500	70	Lee	− 0.08	12	− 2.82	+2.74
1865		Nov. 16.743	262.743	70	Lee	−25.25	12	−23.15	−2.10
1871		Nov. 21.536	267.536	152	F	+ 8.07	11	+ 6.88	+1.19
1884		Dec. 4.708	280.708	99	B	+40.18	13	+39.89	+0.29
1899		Dec. 11.561	287.561	124	F	−16.79	12	−11.24	−5.55
1909		Dec. 21.530	297.530	100	Lee	+34.25	16	+29.33	+4.92
1936		Dec. 28.499	304.499	106	Lee, B, F	−28.89	12	−28.56	−0.33
	Mean		265.166						
	Observations 1912–1913								
			2,419	m		km		km	km
I B² 3147	1912	Nov. 8.680	715.680	180	Lee	+ 0.40	13	− 2.88	+3.28
3153		Nov. 18.569	725.569	240	"	+38.57	13	+38.39	+0.18
3174		Nov. 29.535	736.535	130	"	− 7.71	13	− 3.20	−4.51
3184		Dec. 6.522	743.522	196	"	+ 9.33	12	+ 7.11	+2.22
3200		Dec. 23.507	760.507	154	"	+41.83	10	+41.75	+0.08
3213		Dec. 25.511	762.511	174	"	+36.49	12	+33.99	+2.50
3215		Dec. 27.499	764.499	130	"	+21.29	12	+22.62	−1.33
3229	1913	Jan. 8.505	776.505	120	"	−15.98	12	−15.99	+0.01
3235		Jan. 13.510	781.510	120	"	+51.44	12	+49.77	+1.67
3243		Jan. 20.538	788.538	139	"	+ 9.36	12	+ 5.40	+3.96
	Mean		755.538						

Elements of the orbit.—With the foregoing data and the Lehmann-Filhés method, I computed three orbits corresponding to the following seasons: (*a*) 1906 and 1907; (*b*) 1908; (*c*) 1912–1913.[1]

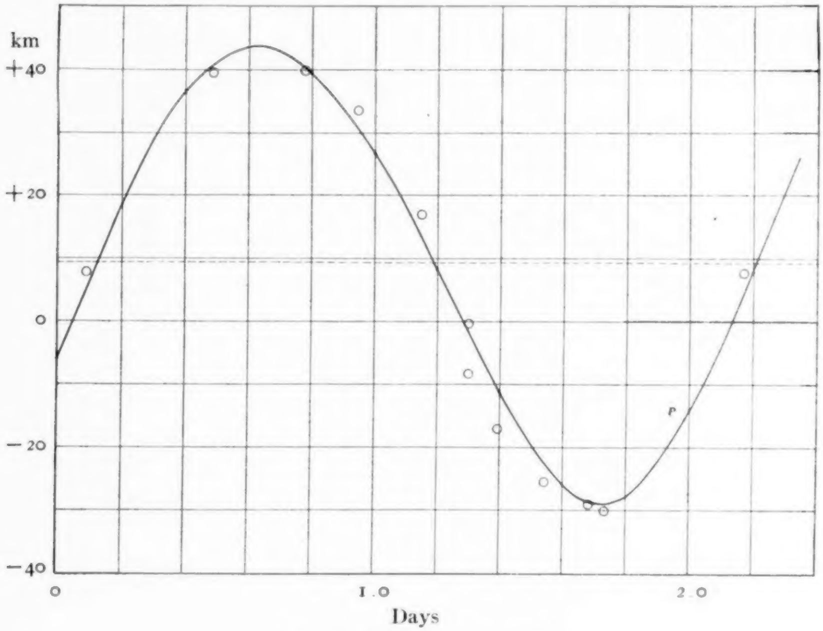


FIG. 2.—Velocity-curve, 1908

The relatively small number of plates taken during the years 1906 and 1907 did not permit us to separate them in two groups and to compute the orbit for each of those years. The elements are almost identical with those obtained by Fox from those plates. The elements of those three orbits are the following:

	1906–1907	1908	Preliminary 1912–1913	Definitive 1912–1913
P	2^d0818	2^d0810	2^d0850	$2^d0866 \pm 0^d0006$
ω	$222^\circ7$	$230^\circ6$	$287^\circ4$	$285^\circ02 \pm 17^\circ2$
e	0.09	0.07	0.11	0.105 ± 0.038
μ	$172^\circ9276$	$172^\circ9667$	$172^\circ66$	$172^\circ53088 \pm 0^\circ094$
T J.D.	2417484.494	2418265.260	2419716.581	2419716.5706 ± 0^d098
$a \sin i$	1,037,262 km	1,036,214	946,040	$975{,}000 \pm 34{,}000$
K	36.4 km/sec.	36.3	33.2	34.18 ± 1.24
γ	+10.3 km/sec.	+ 9.0	+16.1	+17.21

[1] Two of the plates taken in 1908, I B 1893 and I B 1942 (not included in the foregoing tables), are omitted on account of their discrepancies.

The method of least squares was applied for improving the third orbit (1912–1913) and for obtaining some idea about the uncertainty of the elements. The results are given in the fifth column.

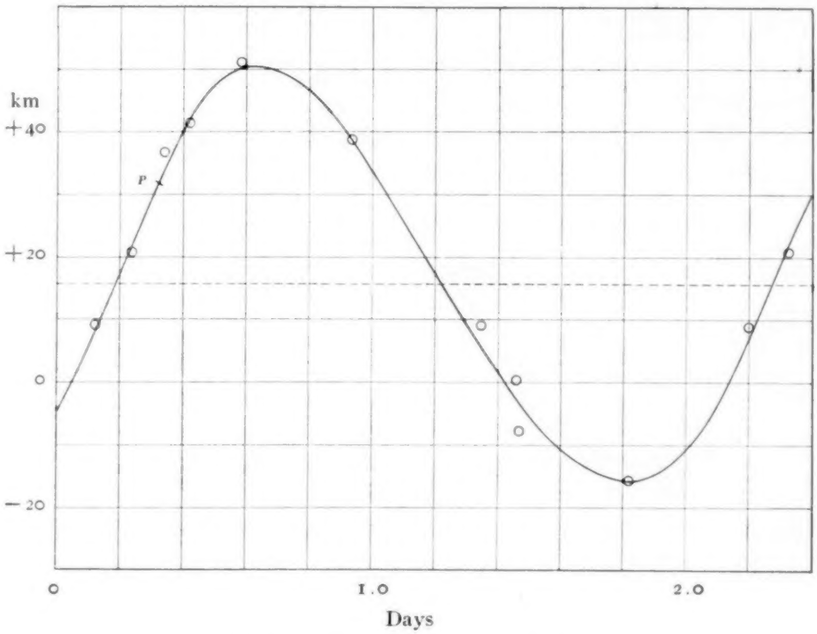


FIG. 3.—Velocity-curve, 1912–1913

Observations were attempted in 1919–1920 by the author for the sake of computing a fourth orbit of the spectroscopic binary, but only four plates—not enough—could be secured. The radial velocities found by measuring those plates are collected in the following table:

Plate	Date (G.M.T.)	Julian Day	Velocity	Computed Velocity
			km	km
I B 5645	1919 December 15.508	2422308.508	+19.85	+52.47
I B 5649	1919 December 19.485	312.485	+14.21	+45.66
I B 5655	1919 December 26.493	319.493	+44.94	+22.84
I B 5661	1920 January 2.517	326.517	−20.27	−12.41

Although there is a continuous change of the elements of the orbit of the spectroscopic binary on account of perturbations, I have tried by using the corrected elements of the orbit for 1912–1913 to compute an ephemeris for the dates of the above-mentioned four plates.

The results of these computations are given in the column headed "Computed Velocity" of the foregoing table. The differences between the measured and the computed radial velocities show very clearly the variation of the elements of the orbit of the spectroscopic binary.

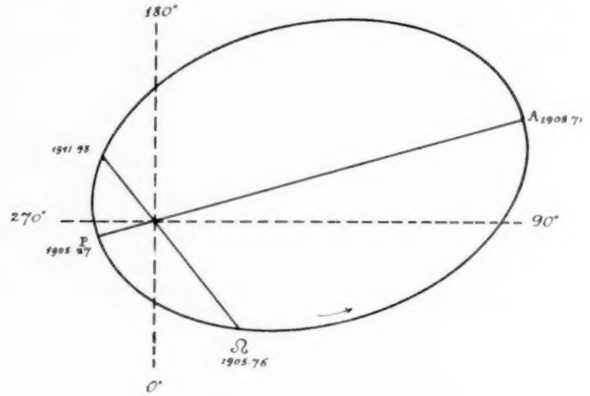


FIG. 4.—True orbit of the bright component around the center of gravity of the visual system.

Change of γ value and determination of the sign of the inclination i of the orbit of the visual system.—The variation of γ during the above-mentioned years gives us the means of finding the sign of the inclination of the visual orbit ($i = \pm 53\overset{\circ}{.}45$).

As is well known, the elements of the visual orbit refer to the motion of the faint component around the bright one (i.e., around the spectroscopic binary). If we consider the orbit of the bright component around the center of gravity of the system (Fig. 4), we find that at the nodal point (Ω), γ has a small value; the bright

component is moving toward the earth and therefore the sign of the inclination of the visual orbit must be *positive*. Now, let:

V = velocity of the faint component of the visual system relatively to the spectroscopic binary;
γ_0 = the radial velocity of the center of gravity of the visual system;
γ = the radial velocity of the center of gravity of the spectroscopic binary;
m = the mass of the faint component of the visual system, and
M = the total mass of the spectroscopic binary.

Then

$$\gamma=\gamma_0-V\frac{m}{M+m}.$$

We also have (Lehmann-Filhés formula[1]):

$$\pi''V=[1.47372]\cdot\frac{a}{P}\sin i\sec\phi[\sin\phi\cos\omega+\cos(v+\omega)],$$

or

$$\pi''V=[1.47372]\frac{a\sin i}{P\sqrt{1-e^2}}[e\cos\omega+\cos(v+\omega)],$$

π'' being the parallax of the system, and i positive, as found above. Hence

$$\gamma=\gamma_0-\frac{m}{M+m}\cdot\frac{1}{\pi''}\left[[1.47372]\frac{a\sin i}{P\sqrt{1-e^2}}(e\cos\omega+\cos[v+\omega])\right] \quad (1)$$

Let

$$\left[[1.47372]\frac{a\sin i}{P\sqrt{1-e^2}}(e\cos\omega+\cos[v+\omega])\right]=N$$

and

$$\frac{m}{M+m}\cdot\frac{1}{\pi''}=C$$

Then

$$\gamma=\gamma_0-CN.$$

Now, suppose we apply this formula to the γ value of each of the above-mentioned three series of observations. Of course the

[1] *Astronomische Nachrichten*, **139**, 308, 1896.

γ value corresponds to the mean date of each series, for which we also have to compute the value of N. We obtain, then, the following equations:

$$\gamma_0 + C \times 0.43079 = 10.3 \text{ km/sec.}$$
$$\gamma_0 + C \times 0.08905 = 9.0$$
$$\gamma_0 + C \times 0.8078 = 17.2$$

which, solved by the method of least squares, give:

$$\gamma_0 = +7.134 \text{ km/sec.}$$
$$C = +11.372.$$

(The value of $C = \frac{1}{\pi''} \cdot \frac{m}{M+m}$ must, of course, always be positive. Thus we have another check upon the sign of i for the visual orbit.) Having the values of γ_0 and C, we can find, by means of the same formula

$$\gamma = \gamma_0 - NC,$$

the values of γ corresponding to the beginning and the end of each series of observations. Thus we find for γ:

First Series	Second Series	Third Series
1906.75, 14.1 km	1908.69, 8.7 km	1912.86, 16.0 km
1907.93, 10.5	1908.99, 7.9	1913.05, 16.6

We see from this table that in the case of the first series, in which there is a difference of 1.18 years between the first and the last observation, the change in γ is about 3.6 km/sec.

Range of γ.—According to the formula (1) the greatest value of γ corresponds to $v+\omega=0$ and the lowest to $v+\omega=180°$ ($\omega=66°.8$). Substituting these two values in (1) we obtain:

Greatest value of $\gamma = +24.9$ km/sec.
Lowest value of $\gamma = +6.1$ km/sec.

Parallax of the system.—The spectrum of 13 Ceti does not give the means of determining its parallax. Nevertheless, an upper

limit of the parallax of the system may be found as follows: Having the value of $C = +11.372$, we obtain

$$\frac{m}{m+M} = 11.372\,\pi \text{ or } \pi = \frac{1}{11.372} \cdot \frac{m}{m+M}$$

and

$$\pi = 0''.088 \frac{m}{m+M} .$$

Hence we see that the parallax can never reach the value of 0″.088 and is very likely a great deal smaller than this. As my present work was just finished the first direct determination of the parallax was published by J. A. Miller,[1]

$$\pi \text{ (relative) } 0''.048 \pm 0''.010,$$

which corresponds to an absolute value of about 0″.050. The corresponding mass-ratio would be:

$$\frac{m}{m+M} = \frac{0.050}{0.088} \text{ or } 57 \text{ per cent}$$

and

$$\frac{m}{M} = 1.32.$$

The fainter visual component would be consequently at least as massive as the bright one.

SUMMARY

Perturbations of the orbit.—Comparing the elements of the three orbits of the spectroscopic binary, we deduce:

1. That the period is shorter near apastron (1908) than near periastron (1912–1913). As is well known, the same perturbation is observed in the motion of the moon around the earth.

2. The eccentricity shows no appreciable variation.

3. The value of $a \sin i$ shows a slight variation which may be attributed to a variation of the major axis, or to a variation of the inclination of the orbit, or to both of these.

4. The motion of the line of apsides is well established. The amount of the precessional motion shows that the mass of the

[1] *Proceedings of American Philosophical Society*, **59**, 87, 1920.

fainter component of the visual system must bear no small ratio to the mass of the spectroscopic binary in order to produce so great a perturbation.

It is, indeed, not improbable that the mass of the fainter visual component may be as great as that of the spectroscopic binary or even greater.[1]

In the course of this investigation the author has enjoyed the interest and encouragement of Professor Edwin B. Frost, the director of the Yerkes Observatory, and of Professor George Van Biesbroeck, and desires to express his appreciation and gratitude.

YERKES OBSERVATORY
February 1920

[1] In the case of 85 Pegasi, which is in all probability a triple system like 13 Ceti, Professor G. Van Biesbroeck has deduced that the mass of the brighter component can be only 0.36 of the mass of the whole system (*Astronomical Journal*, **29**, 173, 1916). Unfortunately, in the case of 13 Ceti, both components are very close, precluding the determination of the mass-ratio.

BRIGHTNESS OF THE NIGHT SKY

By GAVIN J. BURNS

ABSTRACT

Brightness of the night sky; a discussion of the various determinations.—Although the large discrepancies indicate that some unknown sources of error have not been eliminated, the author suggests that close agreement can hardly be expected because of the individual peculiarities of observers' eyes, and because of the effect of varying atmospheric conditions.

Referring to the paper on the foregoing subject in the number of the *Astrophysical Journal* for December,[1] I find that the author calls attention to the great discrepancies that occur in different estimates that have been made of the actual brightness of the non-galactic sky.

The following table contains the results which have been obtained by various observers. It repeats the information contained in Table XII of the aforementioned paper,[2] with some additions.

Authority	Year of Observations	Place	Brightness per Square Degree
1. Newcomb, *Astrophysical Journal*, **14**, 279, 1901	1901	Cape Breton Island	0.029
2. Burns, *ibid.*, **16**, 166, 1902	1899	Weymouth, England	0.050
3. Townley, *Publications of Astronomical Society of Pacific*, **15**, 13, 1903	1902	Lick Observatory	0.050
4. Fabry, *Comptes Rendus*, **150**, 272, 1910	1910?	....	0.036
5. Yntema, *Groningen Publications*, No. 22	1907–1908	Holland	0.140
6. Abbot, *Astronomical Journal*, **27**, 20, 1911	1909–1910	Mount Whitney	0.075
7. Van Rhijn, *Astrophysical Journal*, **50**, 347, 1919	1913	Mount Wilson	0.130
8. Burns, *Journal of British Astronomical Association*, **24**, 463, 1914	1914	Surrey, England	0.030

It will be seen that the highest result is more than four times the lowest, and that the differences are much greater than are likely to arise from casual errors of observation. Moreover, the

[1] *Astrophysical Journal*, **50**, 356, 1919.

[2] *Ibid.*, p. 374.

observed brightness does not appear to depend on either the year or the place of observation. If, for instance, we compare No. 1 with No. 8, and No. 5 with No. 7, we see that European values and American values differ very little from each other. The following remarks are offered as to the probable origin of the discrepancies:

1. Newcomb describes his observations as a "rude attempt." The result has no pretensions to scientific accuracy. He had, however, the advantage of a sky perfectly free from any suspicion of illumination by artificial light, while the air was as clear and pure as any he had ever seen. The method employed was a direct comparison of the light of the sky with the out-of-focus disk of a star.

2. This was the first attempt (to the best of my knowledge) to *measure* the light of the night sky. Previous estimates seem to have been little more than guesses. The circumstances in which the observations were made were not favorable to accuracy. The illumination of the atmosphere by the street lamps of the adjoining town introduced an error of unknown magnitude. The clearness of the air was much less than that of Cape Breton Island. Both these causes would tend to give too high a result. There was, besides, a difficulty owing to the fact that the telescope used gave a yellow tint to any object seen through it. The method adopted was in principle the same as Newcomb's, viz., the comparison of the sky with the out-of-focus disk of a star.

3 and 4. Townley and Fabry both used photographic methods which cannot be expected to give the same results as visual methods. It will be seen, however, that the photographic determinations lie within the same limits as Nos. 1 and 2.

5, 6, and 7. All these determinations were made by the same instrument and presumably by the same method. The method of procedure, as described by Yntema, consists in the following four steps: (*a*) A cardboard box having a circular window at one end fitted with an opal and blue glass is illuminated by an electric lamp contained in the box. At night the cardboard box is mounted on a stand on the roof of a house and the magnitude of the artificial star is determined by comparison with a star of known magnitude by means of a Zoellner photometer. (*b*) The brightness of the artificial star is reduced by a

neutral-tinted absorbing glass, the coefficient of absorption of which was measured. (*c*) The cardboard box with the absorbing glass in front of the window is placed close to the photometer (which is of special construction) and the brightness of the window is measured. (*d*) The brightness of the sky is measured by the same photometer. It will be seen that an error in any of these four measurements will enter into the final result. The chance of error is consequently much greater than in the other determinations enumerated above.

8. This determination was made with much greater care than No. 2. A specially constructed photometer was used. The result was obtained in two steps: (*a*) The sky was viewed through a circular hole in a white screen illuminated by a small electric lamp fed by a current kept at constant strength. The light from the lamp passed through a variable aperture filled in with opal glass having in front a piece of celluloid stained blue so that the illuminated screen might be of the same color as the sky. The area of the aperture gave the intensity of the light from the sky on an arbitrary scale. (*b*) The next step was to determine the constant of the instrument. This was done by a determination of the intensity of the light of the out-of-focus image of a star of known magnitude. The polar sky was the portion selected for comparison. The brightness of the polar sky on the photometric scale was measured on a particularly clear evening in April at a spot many miles from any town where the error caused by artificial lights might be assumed to be negligible. The constant of the instrument was determined from a mean of several observations of Arcturus on different evenings. An obvious defect of this method is that the comparison is made in two distinct stages, which implies a double chance of error.

It seems obvious that there is some undetected source of error in some of the methods employed.

The values obtained by different observers for the absolute amount of light received from one square degree at the North Pole are never likely to be in close agreement. There are two reasons for this discordance.

1. All visual estimates of brightness depend entirely on the eye of the observer, consequently any defect or peculiarity in the

eyesight will make itself felt in the results of eye-observations. As an example of this, take Newcomb's assumption that the "absolute brightness of any object observed with a pencil of rays filling the whole diameter of the pupil varies as the square of that diameter."[1] This statement is probably true of a normal eye, but is certainly not true of my own eyes, in which the central rays of a pencil are more effective in exciting the sensation of light than the outside rays.

2. Another cause that will always make the discordances between individual observations very large is the difference in atmospheric conditions when the observations are made. A slight mistiness in the air immediately increases the apparent luminosity of the sky while it diminishes the apparent brightness of the stars. It consequently happens that on nights differing apparently very little in clearness there may be a marked difference in the brightness of the stars when measured by a photometer. As an instance of this, the out-of-focus disk of Vega appeared to have the brightness 38 on the photometer scale on May 13 this year; whereas six nights afterward, on an evening of about the same clearness, at the sime time in the evening, the brightness was 60 on the scale.

Unfortunately, in England there are only about half a dozen nights in the course of the year which are really suitable for observations of this kind.

There is one other point in this connection which it is interesting to notice. According to the material in Van Rhijn's paper, the total amount of light received from all the stars is equal to 1440 stars of magnitude 1.00 on the Harvard visual scale.[2] The number obtained by Chapman, by an entirely different method, is from 900 to 1000.[3]

Blackheath, S.E.
London
July 17, 1920

[1] *Astrophysical Journal*, **14**, 302, 1901.

[2] *Ibid.*, **50**, 374, 1919.

[3] *Monthly Notices*, **74**, 450, 1914.

PREPARATION OF ABSTRACTS

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. Therefore, *the abstract should summarize the information completely and precisely*, and also, in order to enable a reader to tell at a glance what the article is about and to enable an efficient index of the subject-matter of the abstract to be readily prepared, *the abstract should contain a set of subtitles which together form a complete and precise index of the information contained in the article.*

In the preparation of abstracts, authors should be guided by the following rules, which are illustrated by the abstracts appearing in the *Astrophysical Journal* for January and March 1920.[1] First the new information contained in the article should be determined by a careful analysis; then the subtitles should be formulated; and finally the text should be written and checked.

RULES

1. *Material not new* need not be analyzed or described; a valuable summary of previous work, however, should be noted.

2. *The subtitles should together include all the new information;* that is, every measurement, observation, method, improvement of apparatus, suggestion, and theory which is presented by the author as new and of value in itself.

3. *Each subtitle should describe the corresponding information so precisely* that the chance of any investigator's being misled into thinking the article contains the particular information he desires when it does not, or vice versa, may be small. "Zeeman effect for metallic furnace spectra" is too broad unless all metals have been studied, for an investigator may be interested, at the time, in only one metal; but "Infra-red arc spectrum of iron to 3 μ" evidently satisfies this rule. It is particularly desirable that ranges of variation of temperature, wave-length, pressure, etc., be given.

4. *The text should summarize the author's conclusions and should transcribe all numerical results of general interest*, including all that might be looked for in a table of astronomical and physical constants, with an indication of the

[1] The rules and illustrative abstracts were prepared by G. S. Fulcher, of the National Research Council.

accuracy of each. It should give all the information that anyone, not a specialist in the particular field involved, might care to have in his notebook.

5. *The text should be divided into as many paragraphs* as there are distinct subjects concerning which information is given. Parts of subtitles may be scattered through the text but the subject of each paragraph must be indicated at the beginning.

6. *Complete sentences* should be used except in the case of subtitles. The abstract should be made as readable as the necessary brevity will permit.